Përgjigjet

KAPITULLI 1

Kontrolli i njohurive të mëparshme

1 a $2m^2n + 3mn^2$ b $6x^2 - 12x - 10$
2 a 2^8 b 2^4 c 2^6
3 a $3x + 12$ b $10 - 15x$ c $12x - 30y$
4 a 8 b $2x$ c xy
5 a $2x$ b $10x$ c $\frac{5x}{3}$

Ushtrime 1A

1 a x^7 b $6x^5$ c k d $2p^2$
e x f y^{10} g $5x^2$ h p^2
i $2a^3$ j $2p$ k $6a^9$ l $3a^2b^3$
m $27x^8$ n $24x^{11}$ o $63a^{12}$ p $32y^6$
q $4a^6$ r $6a^{12}$

2 a $9x - 18$ b $x^2 + 9x$
c $-12y + 9y^2$ d $xy + 5x$
e $-3x^2 - 5x$ f $-20x^2 - 5x$
g $4x^2 + 5x$ h $-15y + 6y^3$
i $-10x^2 + 8x$ j $3x^3 - 5x^2$
k $4x - 1$ l $2x - 4$
m $9d^2 - 2c$ n $13 - r^2$
o $3x^3 - 2x^2 + 5x$ p $14y^2 - 35y^3 + 21y^4$
q $-10y^2 + 14y^3 - 6y^4$ r $4x + 10$
s $11x - 6$ t $7x^2 - 3x + 7$
u $-2x^2 + 26x$ v $-9x^3 + 23x^2$

3 a $3x^3 + 5x^5$ b $3x^4 - x^6$ c $\frac{x^3}{2} - x$
d $4x^2 + \frac{5}{2}$ e $\frac{7x^6}{5} + x$ f $3x^4 - \frac{5x^2}{3}$

Ushtrime 1B

1 a $x^2 + 11x + 28$
b $x^2 - x - 6$
c $x^2 - 4x + 4$
d $2x^2 + 3x - 2xy - 3y$
e $4x^2 + 11xy - 3y^2$
f $6x^2 - 10xy - 4y^2$
g $2x^2 - 11x + 12$
h $9x^2 + 12xy + 4y^2$
i $4x^2 + 6x + 16xy + 24y$
j $2x^2 + 3xy + 5x + 15y - 25$
k $3x^2 - 4xy - 8x + 4y + 5$
l $2x^2 + 5x - 7xy - 4y^2 - 20y$
m $x^2 + 2x + 2xy + 6y - 3$
n $2x^2 + 15x + 2xy + 12y + 18$
o $13y - 4x + 12 - 4y^2 + xy$
p $12xy - 4y^2 + 3y + 15x + 10$
q $5xy - 20y - 2x^2 + 11x - 12$
r $22y - 4y^2 - 5x + xy - 10$

2 a $5x^2 - 15x - 20$
b $14x^2 + 7x - 70$
c $3x^2 - 18x + 27$
d $x^3 - xy^2$
e $6x^3 + 8x^2 + 3x^2y + 4xy$
f $x^2y - 4xy - 5y$
g $12x^2y + 6xy - 8xy^2 - 4y^2$
h $19xy - 35y - 2x^2y$
i $10x^3 - 4x^2 + 5x^2y - 2xy$
j $x^3 + 3x^2y - 2x^2 + 6xy - 8x$
k $2x^2y + 9xy + xy^2 + 5y^2 - 5y$
l $6x^2y + 4xy^2 + 2y^2 - 3xy - 3y$
m $2x^3 + 2x^2y - 7x^2 + 3xy - 15x$
n $24x^3 - 6x^2y - 26x^2 + 2xy + 6x$
o $6x^3 + 15x^2 - 3x^2y - 18xy^2 - 30xy$
p $x^3 + 6x^2 + 11x + 6$
q $x^3 + x^2 - 14x - 24$
r $x^3 - 3x^2 - 13x + 15$
s $x^3 - 12x^2 + 47x - 60$
t $2x^3 - x^2 - 5x - 2$
u $6x^3 + 19x^2 + 11x - 6$
v $18x^3 - 15x^2 - 4x + 4$
w $x^3 - xy^2 - x^2 + y^2$
x $8x^3 - 36x^2y + 54xy^2 - 27y^3$

3 $2x^2 - xy + 29x - 7y + 24$
4 $4x^3 + 12x^2 + 5x - 6\,cm^3$
5 $a = 12,\ b = 32,\ c = 3,\ d = -5$

Sfidë
$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Ushtrime 1C

1 a $4(x + 2)$ b $6(x - 4)$
c $5(4x + 3)$ d $2(x^2 + 2)$
e $4(x^2 + 5)$ f $6x(x - 3)$
g $x(x - 7)$ h $2x(x + 2)$
i $x(3x - 1)$ j $2x(3x - 1)$
k $5y(2y - 1)$ l $7x(5x - 4)$
m $x(x + 2)$ n $y(3y + 2)$
o $4x(x + 3)$ p $5y(y - 4)$
q $3xy(3y + 4x)$ r $2ab(3 - b)$
s $5x(x - 5y)$ t $4xy(3x + 2y)$
u $5y(3 - 4z^2)$ v $6(2x^2 - 5)$
w $xy(y - x)$ x $4y(3y - x)$

2 a $x(x + 4)$ b $2x(x + 3)$
c $(x + 8)(x + 3)$ d $(x + 6)(x + 2)$
e $(x + 8)(x - 5)$ f $(x - 6)(x - 2)$
g $(x + 2)(x + 3)$ h $(x - 6)(x + 4)$
i $(x - 5)(x + 2)$ j $(x + 5)(x - 4)$
k $(2x + 1)(x + 2)$ l $(3x - 2)(x + 4)$
m $(5x - 1)(x - 3)$ n $2(3x + 2)(x - 2)$
o $(2x - 3)(x + 5)$ p $2(x^2 + 3)(x^2 + 4)$
q $(x + 2)(x - 2)$ r $(x + 7)(x - 7)$
s $(2x + 5)(2x - 5)$ t $(3x + 5y)(3x - 5y)$
u $4(3x + 1)(3x - 1)$ v $2(x + 5)(x - 5)$
w $2(3x - 2)(x - 1)$ x $3(5x - 1)(x + 3)$

3 a $x(x^2 + 2)$ b $x(x^2 - x + 1)$
c $x(x^2 - 5)$ d $x(x + 3)(x - 3)$
e $x(x - 4)(x + 3)$ f $x(x + 5)(x + 6)$
g $x(x - 1)(x - 6)$ h $x(x + 8)(x - 8)$
i $x(2x + 1)(x - 3)$ j $x(2x + 3)(x + 5)$
k $x(x + 2)(x - 2)$ l $3x(x + 4)(x + 5)$

4 $(x^2 + y^2)(x + y)(x - y)$
5 $x(3x + 5)(2x - 1)$

Sfidë
$(x - 1)(x + 1)(2x + 3)(2x - 3)$

Ushtrime 1D

1 a x^5 b x^{-2} c x^4 d x^3
e x^5 f $12x^0 = 12$ g $3x^{\frac{1}{5}}$ h $5x$
i $6x^{-1}$ j $x^{\frac{5}{6}}$ k $x^{\frac{17}{6}}$ l $x^{\frac{1}{6}}$

2 **a** 5 **b** 729 **c** 3 **d** $\frac{1}{16}$
e $\frac{1}{3}$ **f** $\frac{-1}{125}$ **g** 1 **h** 216
i $\frac{125}{64}$ **j** $\frac{9}{4}$ **k** $\frac{5}{6}$ **l** $\frac{64}{49}$

3 **a** $8x^5$ **b** $\frac{5}{x^2}-\frac{2}{x^3}$ **c** $5x^4$
d $\frac{1}{x^2}+4$ **e** $\frac{2}{x^3}+\frac{1}{x^2}$ **f** $\frac{8}{27}x^6$
g $\frac{3}{x}-5x^2$ **h** $\frac{1}{3x^2}+\frac{1}{5x}$

4 **a** 3 **b** $\frac{16}{\sqrt[3]{x}}$

5 **a** $\frac{x}{2}$ **b** $\frac{32}{x^6}$

Ushtrime 1E

1 **a** $2\sqrt{7}$ **b** $6\sqrt{2}$ **c** $5\sqrt{2}$ **d** $4\sqrt{2}$
e $3\sqrt{10}$ **f** $\sqrt{3}$ **g** $\sqrt{3}$ **h** $6\sqrt{5}$
i $7\sqrt{2}$ **j** $12\sqrt{7}$ **k** $-3\sqrt{7}$ **l** $9\sqrt{5}$
m $23\sqrt{5}$ **n** 2 **o** $19\sqrt{3}$

2 **a** $2\sqrt{3}+3$ **b** $3\sqrt{5}-\sqrt{15}$
c $4\sqrt{2}-\sqrt{10}$ **d** $6+2\sqrt{5}-3\sqrt{2}-\sqrt{10}$
e $6-2\sqrt{7}-3\sqrt{3}+\sqrt{21}$ **f** $13+6\sqrt{5}$
g $8-6\sqrt{3}$ **h** $5-2\sqrt{3}$
i $3+5\sqrt{11}$

3 $3\sqrt{3}$

Ushtrime 1F

1 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$ **c** $\frac{\sqrt{2}}{2}$
d $\frac{\sqrt{5}}{5}$ **e** $\frac{1}{2}$ **f** $\frac{1}{4}$
g $\frac{\sqrt{13}}{13}$ **h** $\frac{1}{3}$

2 **a** $\frac{1-\sqrt{3}}{-2}$ **b** $\sqrt{5}-2$ **c** $\frac{3+\sqrt{7}}{2}$
d $3+\sqrt{5}$ **e** $\frac{\sqrt{5}+\sqrt{3}}{2}$ **f** $\frac{(3-\sqrt{2})(4+\sqrt{5})}{11}$
g $5(\sqrt{5}-2)$ **h** $5(4+\sqrt{14})$ **i** $\frac{11(3-\sqrt{11})}{-2}$
j $\frac{5-\sqrt{21}}{-2}$ **k** $\frac{14-\sqrt{187}}{3}$ **l** $\frac{35+\sqrt{1189}}{6}$
m -1

3 **a** $\frac{11+6\sqrt{2}}{49}$ **b** $9-4\sqrt{5}$ **c** $\frac{44+24\sqrt{2}}{49}$
d $\frac{81-30\sqrt{2}}{529}$ **e** $\frac{13+2\sqrt{2}}{161}$ **f** $\frac{7-3\sqrt{3}}{11}$

4 $-\frac{7}{4}+\frac{\sqrt{5}}{4}$

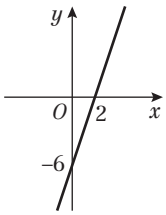
KAPITULLI 2

Kontrolli i njohurive të mëparshme

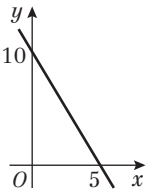
1 **a** $x=-5$ **b** $x=3$
c $x=5$ ose $x=-5$ **d** 16 ose 0

2 **a** $(x+3)(x+5)$ **b** $(x+5)(x-2)$
c $(3x+1)(x-5)$ **d** $(x-20)(x+20)$

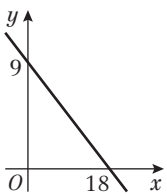
3 **a**



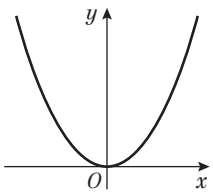
b



3 **c**



d



4 **a** $x<3$ **b** $x \perp 9$ **c** $x :: 2.5$ **d** $x>-7$

Ushtrime 2A

1 **a** $x=-1$ ose $x=-2$ **b** $x=-1$ ose $x=-4$
c $x=-5$ ose $x=-2$ **d** $x=3$ ose $x=-2$
e $x=3$ ose $x=5$ **f** $x=4$ ose $x=5$
g $x=6$ ose $x=-1$ **h** $x=6$ ose $x=-2$

2 **a** $x=0$ ose $x=4$ **b** $x=0$ ose $x=25$
c $x=0$ ose $x=2$ **d** $x=0$ ose $x=6$
e $x=-\frac{1}{2}$ ose $x=-3$ **f** $x=-\frac{1}{3}$ ose $x=\frac{3}{2}$
g $x=-\frac{2}{3}$ ose $x=\frac{3}{2}$ **h** $x=\frac{3}{2}$ ose $x=\frac{5}{2}$

3 **a** $x=\frac{1}{3}$ ose $x=-2$ **b** $x=3$ ose $x=0$
c $x=13$ ose $x=1$ **d** $x=2$ ose $x=-2$
e $x=\pm\frac{\sqrt{5}}{3}$ **f** $x=3\pm\sqrt{13}$
g $x=\frac{1\pm\sqrt{11}}{3}$ **h** $x=1$ ose $x=-\frac{7}{6}$
i $x=-\frac{1}{2}$ ose $x=\frac{7}{3}$ **j** $x=0$ ose $x=-\frac{11}{6}$

4 $x=4$

5 $x=-1$ ose $x=-\frac{2}{25}$

Ushtrime 2B

1 **a** $x=\frac{1}{2}(-3\pm\sqrt{5})$ **b** $x=\frac{1}{2}(3\pm\sqrt{17})$
c $x=-3\pm\sqrt{3}$ **d** $x=\frac{1}{2}(5\pm\sqrt{33})$
e $x=\frac{1}{3}(-5\pm\sqrt{31})$ **f** $x=\frac{1}{2}(1\pm\sqrt{2})$
g $x=2$ ose $x=-\frac{1}{4}$ **h** $x=\frac{1}{11}(-1\pm\sqrt{78})$

2 **a** $x=-0.586$ ose $x=-3.41$ **b** $x=7.87$ ose $x=0.127$
c $x=0.765$ ose $x=-11.8$ **d** $x=8.91$ ose $x=-1.91$
e $x=0.105$ ose $x=-1.90$ **f** $x=3.84$ ose $x=-2.34$
g $x=4.77$ ose $x=0.558$ **h** $x=4.89$ ose $x=-1.23$

3 **a** $x=-6$ ose $x=-2$ **b** $x=1.09$ ose $x=-10.1$
c $x=9.11$ ose $x=-0.110$ **d** $x=-\frac{1}{2}$ ose $x=-2$
e $x=1$ ose $x=-9$ **f** $x=1$
g $x=4.68$ ose $x=-1.18$ **h** $x=3$ ose $x=5$

4 *Syprina* $=\frac{1}{2}(2x)\,(x+(x+10))=50\,m^2$
Pra, $x^2+5x-25=0$
Nga formula kuadratike:
$x=\frac{1}{2}(-5\pm5\sqrt{5})$

Lartësia $= 2x = 5(\sqrt{5} - 1)$ *m*

Sfidë

$x = 13$

Ushtrime 2C

1 **a** $(x + 2)^2 - 4$ **b** $(x - 3)^2 - 9$
c $(x - 8)^2 - 64$ **d** $(x + \frac{1}{2})^2 - \frac{1}{4}$
e $(x - 7)^2 - 49$

2 **a** $2(x + 4)^2 - 32$ **b** $3(x - 4)^2 - 48$
c $5(x + 2)^2 - 20$ **d** $2(x - \frac{5}{4})^2 - \frac{25}{8}$
e $-2(x - 2)^2 + 8$

3 **a** $2(x + 2)^2 - 7$ **b** $5(x - \frac{3}{2})^2 - \frac{33}{4}$
c $3(x + \frac{1}{3})^2 - \frac{4}{3}$ **d** $-4(x + 2)^2 + 26$
e $-8(x - \frac{1}{8})^2 + \frac{81}{8}$

4 $a = \frac{3}{2}$, $b = \frac{15}{4}$

5 $A = 6$, $B = 0.04$, $C = -10$

Ushtrime 2D

1 **a** $x = -3 \pm 2\sqrt{2}$ **b** $x = -6 \pm \sqrt{33}$
c $x = -2 \pm \sqrt{6}$ **d** $x = 5 \pm \sqrt{30}$

2 **a** $x = \frac{1}{2}(-3 \pm \sqrt{15})$ **b** $x = \frac{1}{5}(-4 \pm \sqrt{26})$
c $x = \frac{1}{8}(1 \pm \sqrt{129})$ **d** $x = \frac{1}{2}(-3 \pm \sqrt{39})$

3 **a** $p = -7$, $q = -48$
b $(x - 7)^2 = 48$
$x = 7 \pm \sqrt{48} = 7 \pm 4\sqrt{3}$
$r = 7$, $s = 4$

4 $x^2 + 2bx + c = (x + b)^2 - b^2 + c$
$(x + b)^2 = b^2 - c$
$x = -b \pm \sqrt{b^2 - c}$

Sfidë

a $ax^2 + 2bx + c = 0$

$x^2 + \frac{2b}{a}x + \frac{c}{a} = 0$

$\left(x + \frac{b}{a}\right)^2 - \frac{b^2}{a^2} + \frac{c}{a} = 0$

$\left(x + \frac{b}{a}\right)^2 = \frac{b^2 - ac}{a^2}$

$x = -\frac{b}{a} \pm \sqrt{\frac{b^2 - ac}{a^2}}$

b $ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ushtrime 2E

1 **a** 8 **b** 7 **c** 3 **d** 10.5 **e** 0
f 0 **g** 25 **h** 2 **i** 7

2 $a = 4$ ose $a = -2$

3 **a** $\frac{2}{3}$ **b** 2 dhe −9 **c** −10 dhe 4
d 12 dhe −12 **e** 0, −5 dhe −7 **f** 0, 3 dhe −8

4 $x = 3$ dhe $x = 2$

5 $x = 0$, 2.5 dhe 6

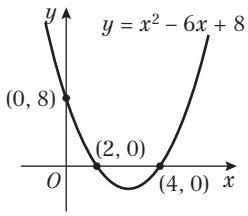
6 **a** $(x - 1)^2 + 1$
$p = -1$, $q = 1$
b Kufizat në katror janë përherë ≥ 0, pra vlera minimale është
$0 + 1 = 1$

7 **a** −2 dhe −1 **b** 2, −2, $2\sqrt{2}$ dhe $-2\sqrt{2}$
c −1 dhe $\frac{1}{3}$ **d** $\frac{1}{2}$ dhe 1
e 4 dhe 25 **f** 8 dhe −27

8 **a** $(3^x - 27)(3^x - 1)$ **b** 0 dhe 3

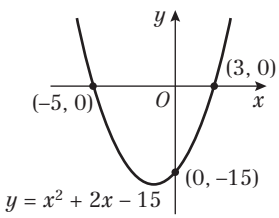
Ushtrime 2F

1 **a**



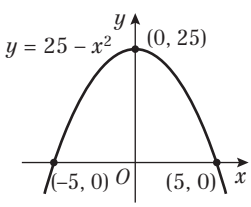
Pika e kthimit: (3, −1)
Drejtëza e simetrisë: $x = 3$

b



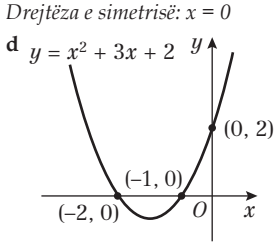
Pika e kthimit: (−1, −16)
Drejtëza e simetrisë: $x = -1$

c



Pika e kthimit:(0, 25)
Drejtëza e simetrisë: $x = 0$

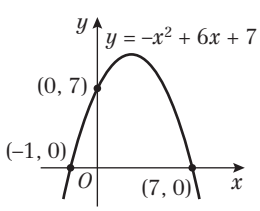
d



Pika e kthimit: $(-\frac{3}{2}, -\frac{1}{4})$

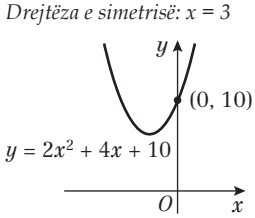
Drejtëza e simetrisë: $x = -\frac{3}{2}$

e



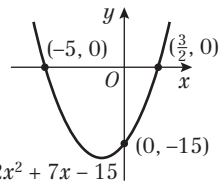
Pika e kthimit: (3, 16)
Drejtëza e simetrisë: $x = 3$

f



Pika e kthimit: (−1, 8)
Drejtëza e simetrisë: x = −1

g

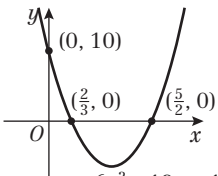


$y = 2x^2 + 7x - 15$

Pika e kthimit: $\left(-\frac{7}{4}, -\frac{169}{8}\right)$

Drejtëza e simetrisë: $x = -\frac{7}{4}$

h

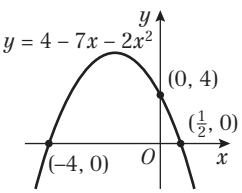


$y = 6x^2 - 19x + 10$

Pika e kthimit: $\left(\frac{19}{12}, -\frac{121}{24}\right)$

Drejtëza e simetrisë: $x = \frac{19}{12}$

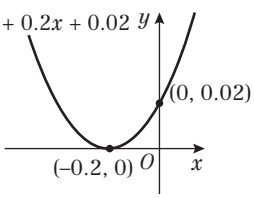
i



Pika e kthimit: $\left(-\frac{7}{4}, \frac{81}{8}\right)$

Drejtëza e simetrisë: $x = -\frac{7}{4}$

j $y = 0.5x^2 + 0.2x + 0.02$



Pika e kthimit: (−0.2, 0)
Drejtëza e simetrisë: x = −0.2

2 **a** $a = 1, b = -8, c = 15$
b $a = -1, b = 3, c = 10$
c $a = 2, b = 0, c = -18$
d $a = \frac{1}{4}, b = -\frac{3}{4}, c = -1$

3 $a = 3, b = -30, c = 72$

Ushtrime 2G

1 **a** **i** 52 **ii** −23 **iii** 37
iv 0 **v** −44
b **i** $h(x)$ **ii** $f(x)$ **iii** $k(x)$
iv $j(x)$ **v** $g(x)$

2 $k < 9$

3 $t = \frac{9}{8}$

4 $s = 4$

5 $k > \frac{4}{3}$

6 **a** $p = 6$ **b** $x = -9$

7 **a** $k^2 + 16$
b k^2 *është përherë pozitiv pra,* $k^2 + 16 > 0$

Sfidë

a *Duhet* $b^2 > 4ac$. *Në qoftë se a, c > 0 ose a, c < 0, zgjidh b të tillë që* $b > \sqrt{4ac}$. *Në qoftë se a > 0 dhe c < 0 (ose anasjelltas), atëherë* $4ac < 0$, *pra* $4ac < b^2$ *për të gjitha b.*

b Jo në qoftë se njëra nga *a* ose *c* është negative sepse kjo do të kërkonte që *b* të ishte rrënja katrore e një numri negativ. E mundshme në qoftë se të dy janë negativë ose të dy janë pozitivë.

KAPITULLI 3

Kontrolli i njohurive të mëparshme

1 **a** $A \cap B = \{1, 2, 4\}$ **b** $(A \cup B)' = \{7, 9, 11, 13\}$

2 **a** $5\sqrt{3}$ **b** $\sqrt{5} + 2\sqrt{2}$

3 **a** *graph* **ii** **b** *graph* **iii** **c** *graph* **i**

Ushtrime 3A

1 **a** $x = 4, y = 2$ **b** $x = 1, y = 3$
c $x = 2, y = -2$ **d** $x = 4\frac{1}{2}, y = -3$
e $x = -\frac{2}{3}, y = 2$ **f** $x = 3, y = 3$

2 **a** $x = 5, y = 2$ **b** $x = 5\frac{1}{2}, y = -6$
c $x = 1, y = -4$ **d** $x = 1\frac{3}{4}, y = \frac{1}{4}$

3 **a** $x = -1, y = 1$ **b** $x = 4, y = -4$
c $x = 0.5, y = -2.5$

4 **a** $3x + ky = 8$ (1); $x - 2ky = 5$ (2)
(1) × 2: $6x + 2ky = 16$ (3)
(2) + (3) $7x = 21$ *pra* $x = 3$
b −2

5 $p = 3, q = 1$

Ushtrime 3B

1 **a** $x = 5, y = 6$ *ose* $x = 6, y = 5$
b $x = 0, y = 1$ *ose* $x = \frac{4}{5}, y = -\frac{3}{5}$
c $x = -1, y = -3$ *ose* $x = 1, y = 3$
d $a = 1, b = 5$ *ose* $a = 3, b = -1$
e $u = 1\frac{1}{2}, v = 4$ *ose* $u = 2, v = 3$
f $x = -1\frac{1}{2}, y = 5\frac{3}{4}$ *ose* $x = 3, y = -1$

2 **a** $x = 3, y = \frac{1}{2}$ *ose* $x = 6\frac{1}{3}, y = -2\frac{5}{6}$
b $x = 4\frac{1}{2}, y = 4\frac{1}{2}$ *ose* $x = 6, y = 3$
c $x = -19, y = -15$ *ose* $x = 6, y = 5$

3 **a** $x = 3 + \sqrt{13}, y = -3 + \sqrt{13}$ *ose* $x = 3 - \sqrt{13}, y = -3 - \sqrt{13}$
b $x = 2 - 3\sqrt{5}, y = 3 + 2\sqrt{5}$ *ose* $x = 2 + 3\sqrt{5}, y = 3 - 2\sqrt{5}$

4 $x = -5, y = 8$ *ose* $x = 2, y = 1$

5 **a** $3x^2 + x(2 - 4x) + 11 = 0$
$3x^2 + 2x - 4x^2 + 11 = 0$
$x^2 - 2x - 11 = 0$
b $x = 1 + 2\sqrt{3}, y = -2 - 8\sqrt{3}$
$x = 1 - 2\sqrt{3}, y = -2 + 8\sqrt{3}$

6 **a** $k = 3, p = -2$
b $x = -6, y = -23$

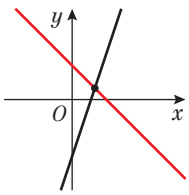
Sfidë

$y = x + k$
$x^2 + (x + k)^2 = 4$

$x^2 + x^2 + 2kx + k^2 - 4 = 0$
$2x^2 + 2kx + k^2 - 4 = 0$ *for one solution* $b^2 - 4ac = 0$
$4k^2 - 4 \times 2(k^2 - 4) = 0$
$4k^2 - 8k^2 + 32 = 0$ $4k^2 = 32$ $k^2 = 8$ $k = \pm 2\sqrt{2}$

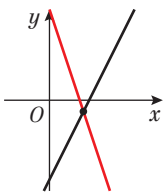
Ushtrime 3C

1 a i



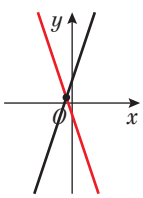
ii *(2, 1)*

b i



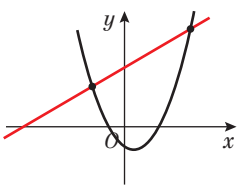
ii *(3, −1)*

c i



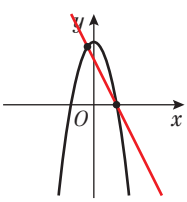
ii *(−0.5, 0.5)*

2 a



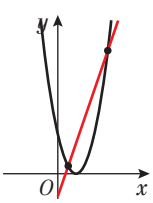
b *(3.5, 9) dhe (−1.5, 4)*

3 a



b *(−1, 8) dhe (3, 0)*

4 a



b *(6, 16) dhe (1, 1)*

5 *(−11, −15) dhe (3, −1)*

6 $(-1\frac{1}{6}, -4\frac{1}{2})$ *dhe (2, 5)*

7 a *2 pikë* **b** *1 pikë* **c** *0 pikë*

8 a $y = 2x - 1$
$x^2 + 4k(2x - 1) + 5k = 0$
$x^2 + 8kx - 4k + 5k = 0$ $x^2 + 8kx + k = 0$

b $k = \frac{1}{16}$ **c** $x = -\frac{1}{4}, y = -\frac{3}{2}$

9 *Në qoftë se notari arrin fundin e pishinës, atëherë*
$0.5x^2 - 3x = 0.3x - 6$
$0.5x^2 - 3.3x + 6 = 0$
$b^2 - 4ac = (-3.3)^2 - 4 \times 0.5 \times 6 = -1.11$
negative, pra nuk ka pika prerje dhe kur zhytet ai nuk e arrin fundin e pishinës.

Ushtrime 3D

1 a $x < 4$ **b** $x \perp 7$ **c** $x > 2\frac{1}{2}$ **d** $x \le -3$
e $x < 11$ **f** $x < 2\frac{3}{5}$ **g** $x > -12$ **h** $x < 1$
i $x \le 8$ **j** $x > 1\frac{1}{7}$

2 a $x \ge 3$ **b** $x < 1$ **c** $x \le -3\frac{1}{4}$ **d** $x < 18$
e $x > 3$ **f** $x \ge 4\frac{2}{5}$ **g** $x < 4$ **h** $x > -7$
i $x \le -\frac{1}{2}$ **j** $x \ge \frac{3}{4}$ **k** $x \ge -\frac{10}{3}$ **l** $x \ge \frac{9}{11}$

3 a $\{x: x > 2\frac{1}{2}\}$ **b** $\{x: 2 < x < 4\}$
c $\{x: 2\frac{1}{2} < x < 3\}$ **d** *Nuk ka vlera*
e $x = 4$ **f** $\{x: x < 1.2\} \cup \{x: x > 2.2\}$
g $\{x: x \le -\frac{2}{3}\} \cup \{x: x \ge \frac{3}{2}\}$

Sfidë

$p = -1, q = 4, r = 6$

Ushtrime 3E

1 a $3 < x < 8$ **b** $-4 < x < 3$
c $x < -2, x > 5$ **d** $x :: -4, x \perp -3$
e $-\frac{1}{2} < x < 7$ **f** $x < -2, x > 2\frac{1}{2}$
g $\frac{1}{2} \le x \le 1\frac{1}{2}$ **h** $x < \frac{1}{3}, x > 2$
i $-3 < x < 3$ **j** $x < -2\frac{1}{2}, x > \frac{2}{3}$
k $x < 0, x > 5$ **l** $-1\frac{1}{2} :: x :: 0$

2 a $-5 < x < 2$ **b** $x < -1, x > 1$
c $\frac{1}{2} < x < 1$ **d** $-3 < x < \frac{1}{4}$

3 a $\{x: 2 < x < 4\}$ **b** $\{x: x > 3\}$
c $\{x: -\frac{1}{4} < x < 0\}$ **d** *Nuk ka vlera*
e $\{x: -5 < x < -3\} \cup \{x: x > 4\}$
f $\{x: -1 < x < 1\} \cup \{x: 2 < x < 3\}$

4 a $x < 0$ *ose* $x > 2$ **b** $x < 0$ *ose* $x > 0.8$
c $x < -1$ *ose* $x > 0$ **d** $x < 0$ *ose* $x > 0.5$
e $x < -\frac{1}{5}$ *ose* $x > \frac{1}{5}$ **f** $x \le -\frac{2}{3}$ *ose* $x \ge 3$

5 a $-2 < k < 6$ **b** $p \le -8$ *ose* $p \ge 0$

6 $\{x: x < -2\} \cup \{x: x > 7\}$

7 a $\{x: x < \frac{2}{3}\}$ **b** $\{x: -\frac{1}{2} < x < 3\}$
c $\{x: -\frac{1}{2} < x < \frac{2}{3}\}$

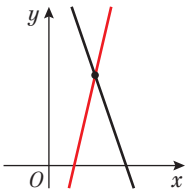
8 $x < 3$ *ose* $x > 5.5$

9 *Nuk ka rrënjë reale kur* $b^2 - 4ac < 0$ $(-2k)^2 - 4 \times k \times 3 < 0$
$4k^2 - 12k = 0$ *kur* $k = 0$ *dhe* $k = 3$
zgjidhja $0 \le k < 3$
vini re se kur $k = 0$ *ekuacioni jep* $3 = 0$

Ushtrime 3F

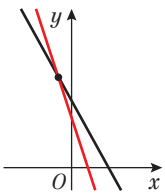
1 **a** $P(3.2, -1.8)$ **b** $x < 3.2$

2 **a** **i**



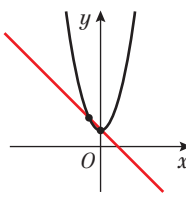
ii $(4, 5)$ **iii** $x \le 4$

b **i**



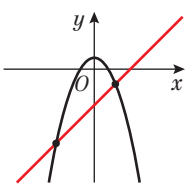
ii $(-3, 23)$ **iii** $x \ge -3$

c **i**



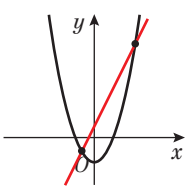
ii $(-2, 9), (0, 5)$ **iii** $-2 \le x \le 0$

d **i**



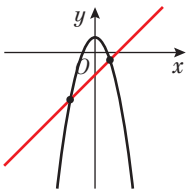
ii $(-5, -22), (3, -6)$ **iii** $x \le -5$ *ose* $x \ge 3$

e **i**



ii $(-2, -1), (9, 76)$ **iii** $-2 \le x \le 9$

f **i**



ii $(-5, -18), (3, -2)$ **iii** $x \le -5$ *ose* $x \ge 3$

3 **a** $-1 < x < 2$ **b** $0.5 < x < 3$

c $x < 0.5$ *ose* $x > 3$ **d** $x < 0$ *ose* $x > 2$

e $1 < x < 3$ **f** $x < -1$ *ose* $x > -0.75$

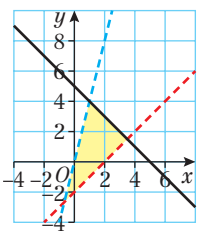
Sfidë

a $(-1.5, -3.75), (6, 0)$

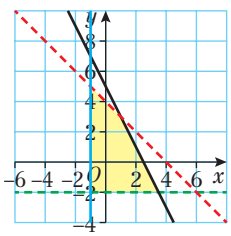
b $\{x: -1.5 < x < 6\}$

Ushtrime 3G

1



2



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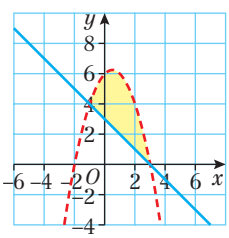

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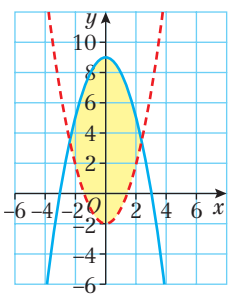

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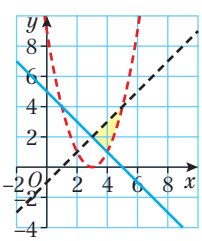
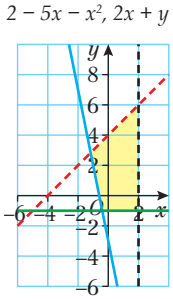


6 **a** $(1, 6), (3, 4), (1, 2)$

b $x \ge 1, y \le 7 - x, y \ge x + 1$

7 $y < 2 - 5x - x^2, 2x + y \perp 0, x + y \le 4$

8 **a**



b $(-\frac{7}{6}, \frac{17}{6}), (2, 6), (2, -1), (-0.4, -1)$

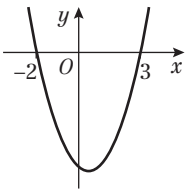
c $(-0.4, -1)$ **d** $\frac{941}{60}$

KAPITULLI 4

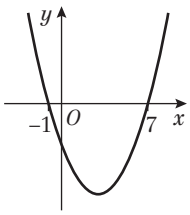
Kontrolli i njohurive të mëparshme

1 **a** $(x + 5)(x + 1)$ **b** $(x - 3)(x - 1)$

2 **a**



b

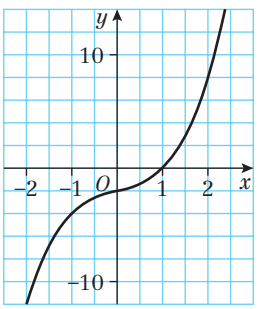


3 **a**

x	−2	−1.5	−1	−0.5	0
y	−12	−6.875	−4	−2.625	−2

x	0.5	1	1.5	2
y	−1.375	0	2.875	8

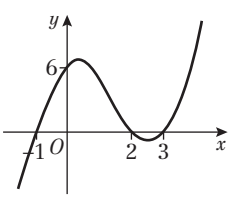
b



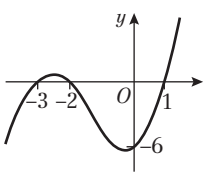
4 **a** $x = 2, y = 4$ **b** $x = 1, y = 1$

Ushtrime 4A

1 **a**



b



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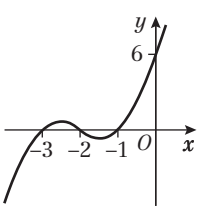

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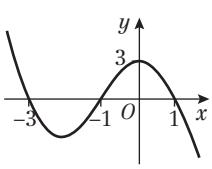
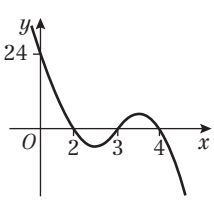


e



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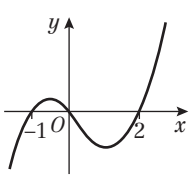
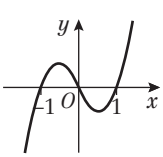
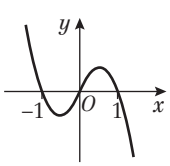


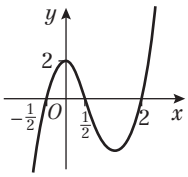
g



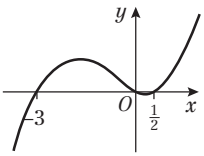
h



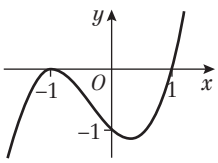
i



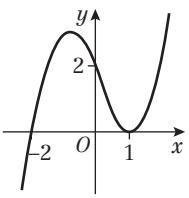
j



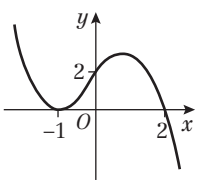
2 **a**



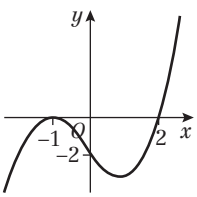
b



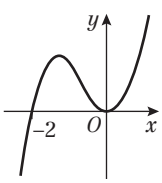
c



d



e



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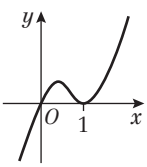
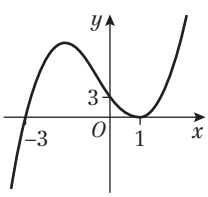


g



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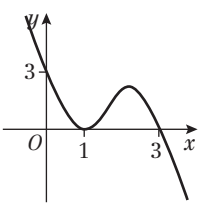
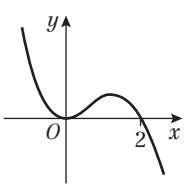
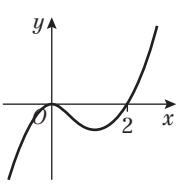


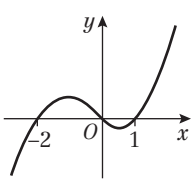
i



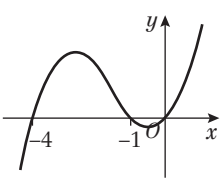
j



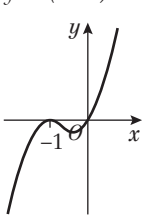
3 **a** $y = x(x + 2)(x - 1)$



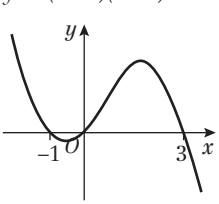
b $y = x(x + 4)(x + 1)$



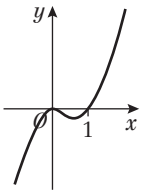
c $y = x(x + 1)^2$



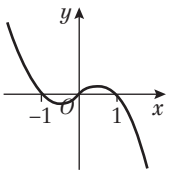
d $y = x(x + 1)(3 - x)$



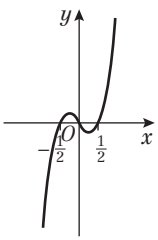
e $y = x^2(x - 1)$



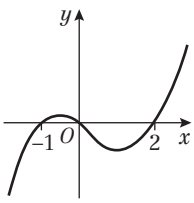
f $y = x(1 - x)(1 + x)$



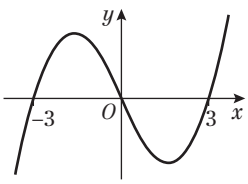
g $y = 3x(2x - 1)(2x + 1)$



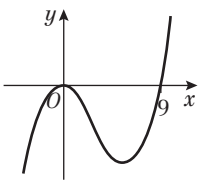
h $y = x(x + 1)(x - 2)$



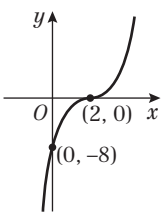
i $y = x(x - 3)(x + 3)$



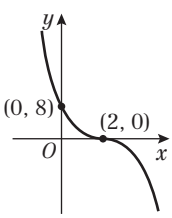
j $y = x^2(x - 9)$



4 **a**



b



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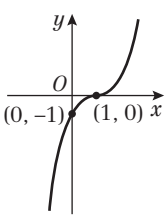
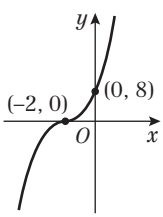
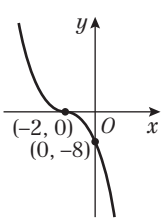


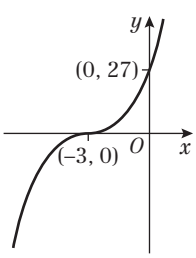
d



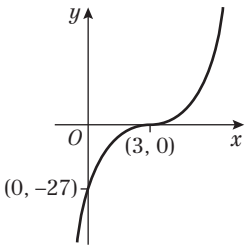
e



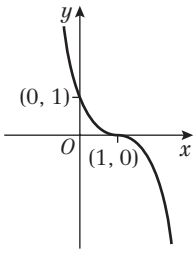
f



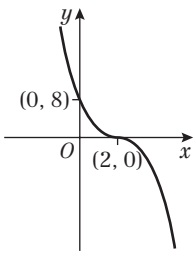
g



h



i



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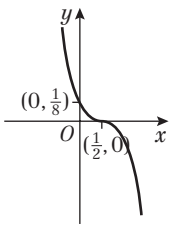
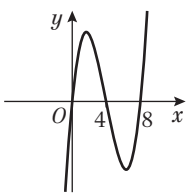


5 **a** $b = 4, c = 1, d = -6$ **b** $(0, -6)$

6 $a = \frac{1}{3}, b = -\frac{4}{3}, c = \frac{1}{3}, d = 2$

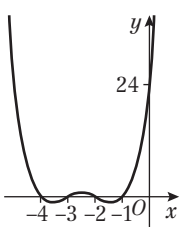
7 **a** $x(x^2 - 12x + 32)$ **b** $x(x - 8)(x - 4)$

c

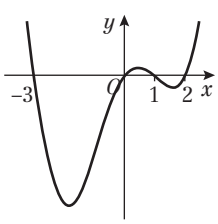


Ushtrime 4B

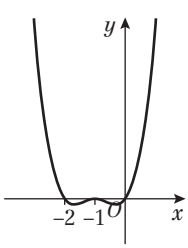
1 **a**



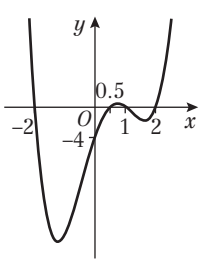
b



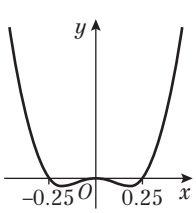
c



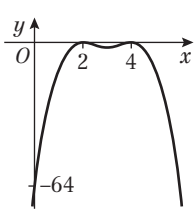
d



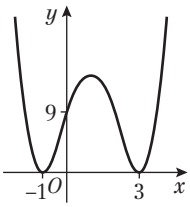
e



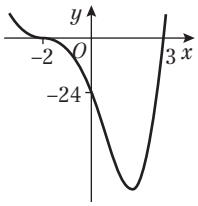
f



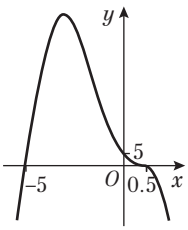
g



h



i



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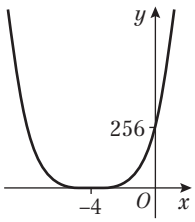
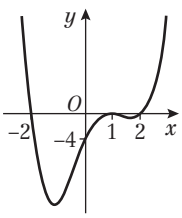
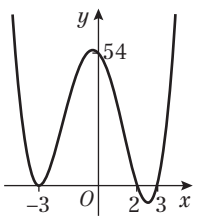


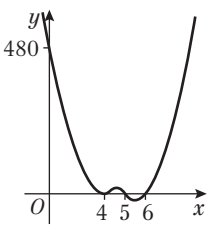
2 a



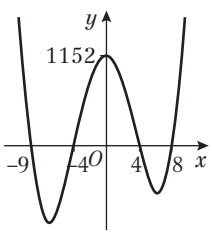
b



c



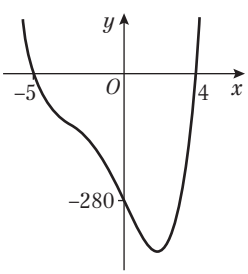
d



3 a *(0, 12)*

b $b = -2$, $c = -7$, $d = 8$, $e = 12$

4

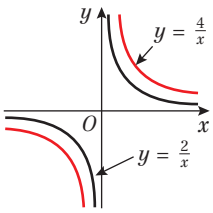


Sfidë

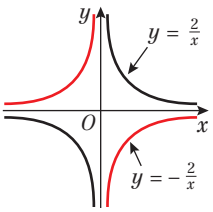
$a = \frac{1}{3}$, $b = -\frac{4}{3}$, $c = -\frac{2}{3}$, $d = 4$, $e = 3$

Ushtrime 4C

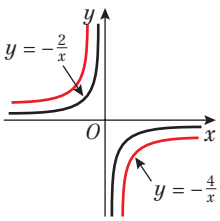
1 a



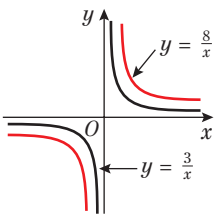
b



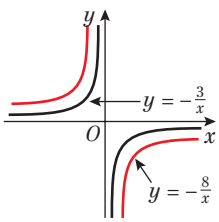
c



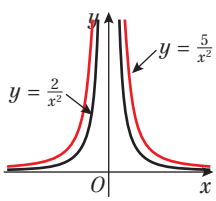
d



e



2 a



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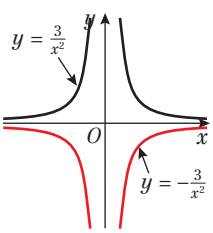

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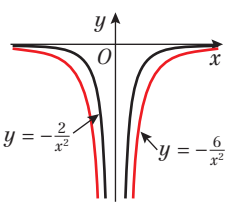
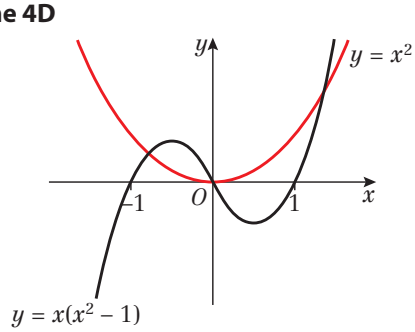


Ushtrime 4D

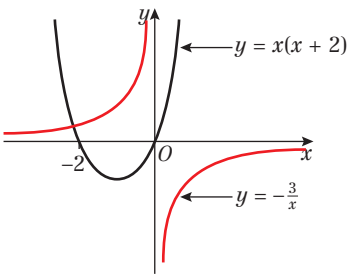
1 a i



ii *3*

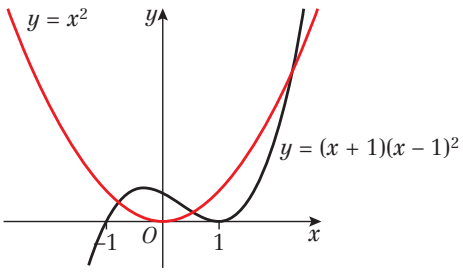
iii $x^2 = x(x^2 - 1)$

b i



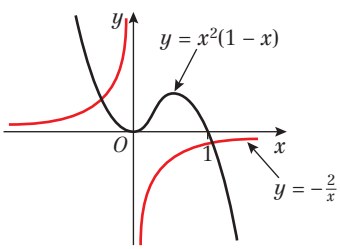
ii *1* **iii** $x(x+2) = -\frac{3}{x}$

c i



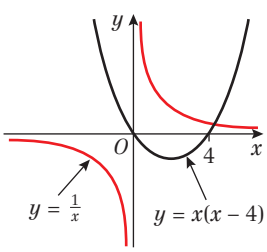
ii *3* **iii** $x^2 = (x+1)(x-1)^2$

d i



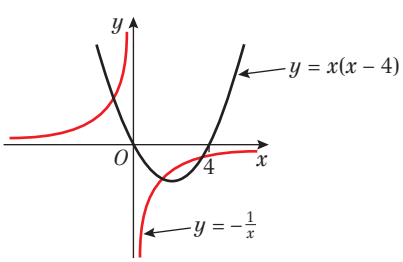
ii *2* **iii** $x^2(1-x) = -\frac{2}{x}$

e i



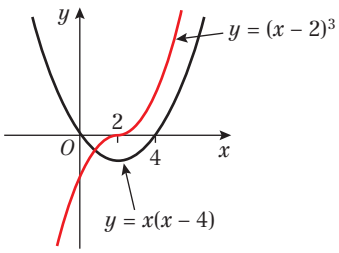
ii *1* **iii** $x(x-4) = \frac{1}{x}$

f i



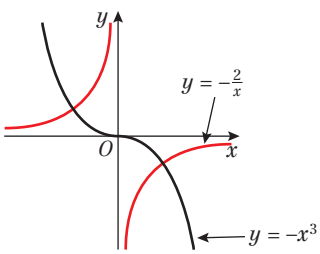
ii *3* **iii** $x(x-4) = -\frac{1}{x}$

g i



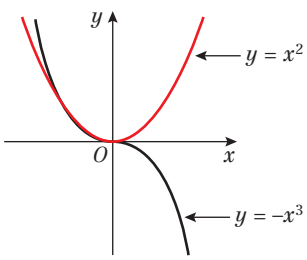
ii *1* **iii** $x(x-4) = (x-2)^3$

h i



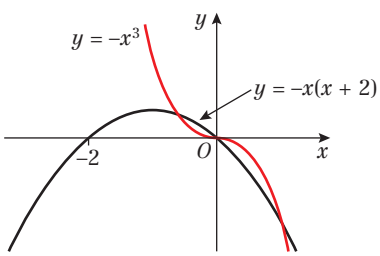
ii *2* **iii** $-x^3 = -\frac{2}{x}$

i i



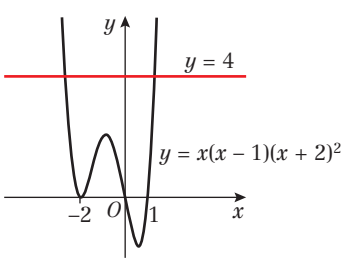
ii *2* **iii** $-x^3 = x^2$

j i



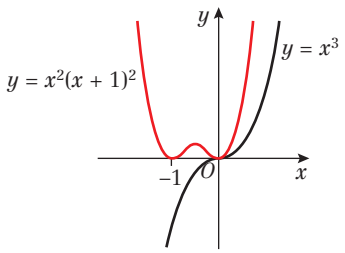
ii *3* **iii** $-x^3 = -x(x+2)$

k i



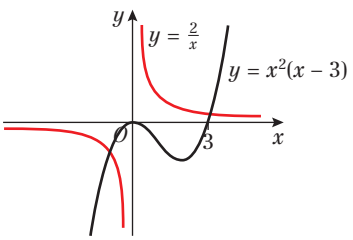
ii *2* **iii** $x(x-1)(x+2)^2 = 4$

1 **i**



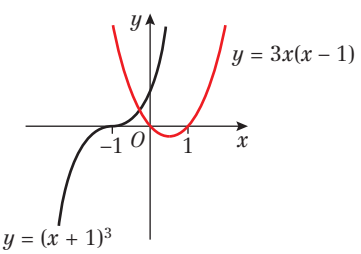
ii *1* **iii** $x^3 = x^2(x + 1)^2$

2 **a**



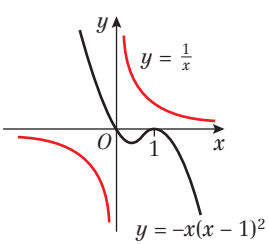
b *Vetëm 2 pika prerje*

3 **a**



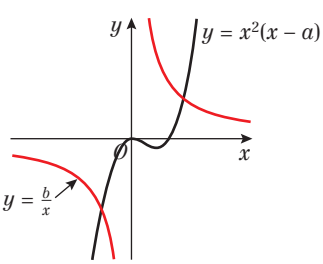
b *Vetëm 1 pikëprerje*

4 **a**



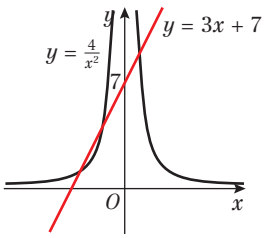
b *Grafikët nuk priten*

5 **a**



b 2: grafikët priten në dy vende, pra ka dy zgjidhje.

6 **a**

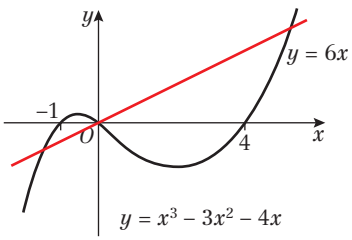


b *3*

c *Hap kllapat dhe thjeshto*

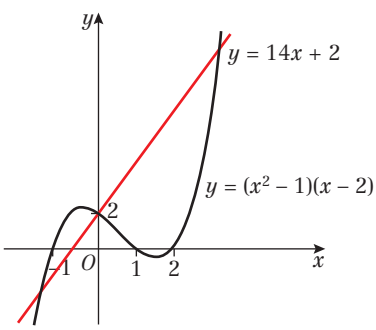
d $(-2, 1), (-1, 4), (\frac{2}{3}, 9)$

7 **a**



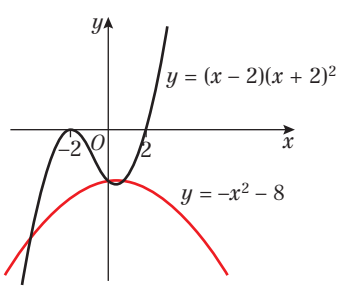
b $(0, 0); (-2, -12); (5, 30)$

8 **a**



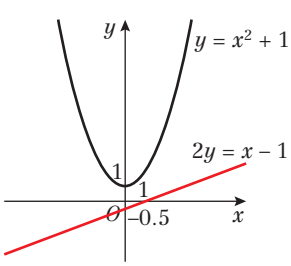
b $(0, 2); (-3, -40); (5, 72)$

9 **a**



b $(0, -8); (1, -9); (-4, -24)$

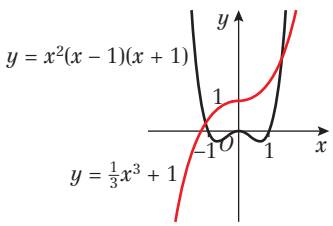
10 **a**



b *Grafikët nuk priten.*

c $a < -\frac{7}{16}$

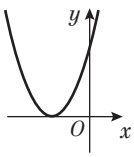
11 **a**



b 2

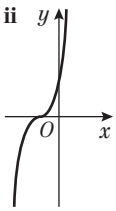
Ushtrime 4E

1 **a** **i**



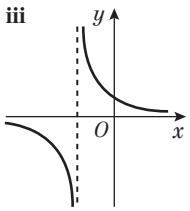
(–2, 0), (0, 4)

ii



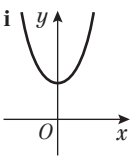
(–2, 0), (0, 8)

iii



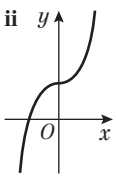
$(0, \frac{1}{2})$, $x = -2$, $y = 0$

b **i**



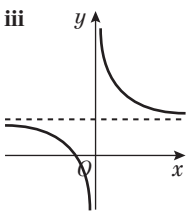
(0, 2)

ii



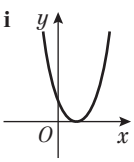
$(-\sqrt[3]{2}, 0)$, (0, 2)

iii



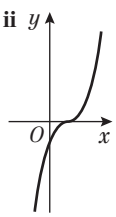
$(-\frac{1}{2}, 0)$, $y = 2$, $x = 0$

c **i**



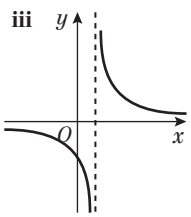
(0, 1), (1, 0)

ii



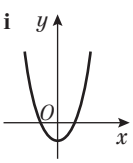
(0, –1), (1, 0)

iii



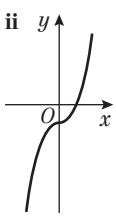
(0, –1), $x = 1$, $y = 0$

d **i**



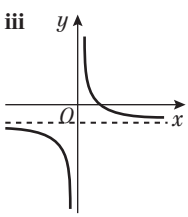
(–1, 0),
(0, –1),
(1, 0)

ii



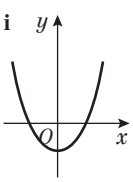
(0, –1), (1, 0)

iii



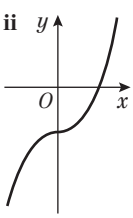
(1, 0), $y = -1$, $x = 0$

e **i**



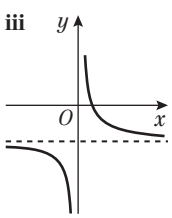
$(-\sqrt{3}, 0)$,
(0, –3),
$(\sqrt{3}, 0)$

ii



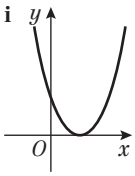
(0, –3),
$(\sqrt[3]{3}, 0)$

iii



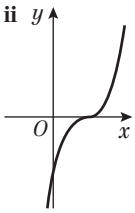
$(\frac{1}{3}, 0)$, $y = -3$, $x = 0$

f **i**



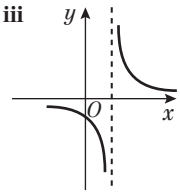
(0, 9), (3, 0)

ii



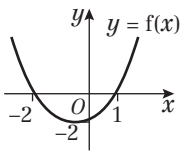
(0, –27), (3, 0)

iii

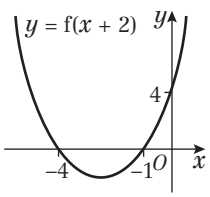


$(0, -\frac{1}{3})$, $x = 3$, $y = 0$

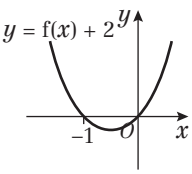
2 **a**



b **i**

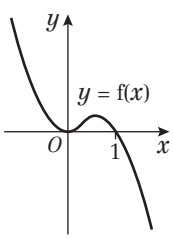


ii

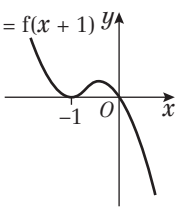


c $f(x + 2) = (x + 1)(x + 4)$; (0, 4)
$f(x) + 2 = (x - 1)(x + 2) + 2$; (0, 0)

3 **a**

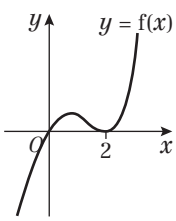


b

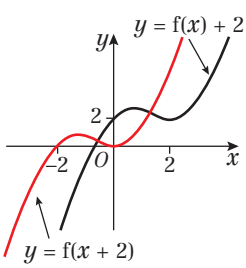


c $f(x + 1) = -x(x + 1)^2$; (0, 0)

4 **a**

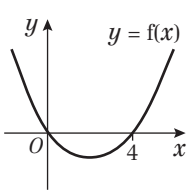


b

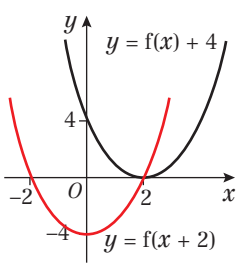


c $f(x + 2) = (x + 2)x^2$; (0, 0); (–2, 0)

5 **a**

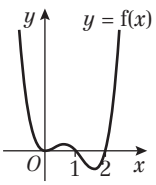


b

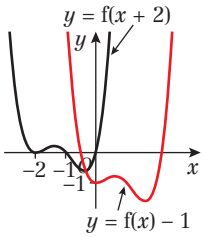


c $f(x + 2) = (x + 2)(x - 2)$; (2, 0); (–2, 0); (0, –4)
$f(x) + 4 = (x - 2)^2$; (2, 0); (0, 4)

6 **a**



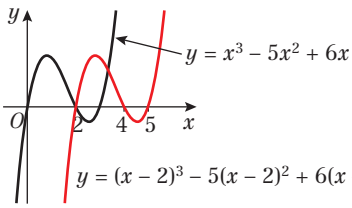
b



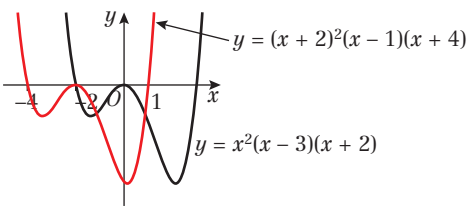
7 **a** *(6, −1)* **b** *(4, 2)*

8 $y = \dfrac{1}{x-4}$

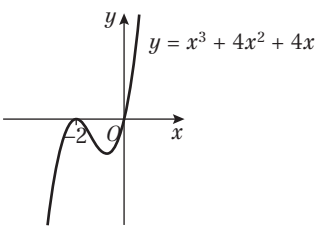
9



10

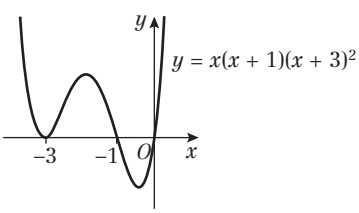


11 **a**



b *−1 ose 1*

12 **a**



b *−2, −3 ose −5*

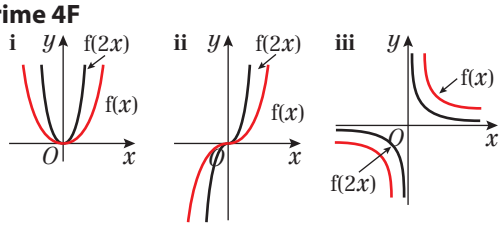
Sfidë

1 *(3, 2)*

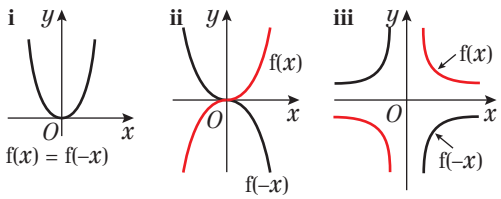
2 **a** *(−7, −12)* **b** *f(x − 2) + 1*

Ushtrime 4F

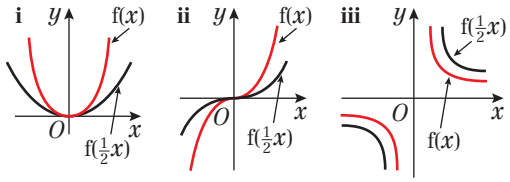
1 **a**



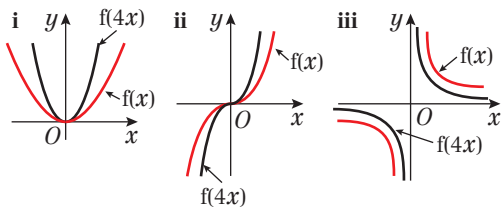
b



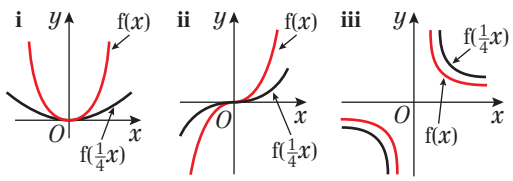
c



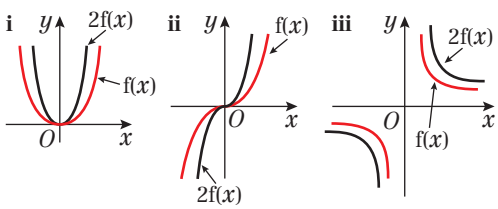
d



e



f



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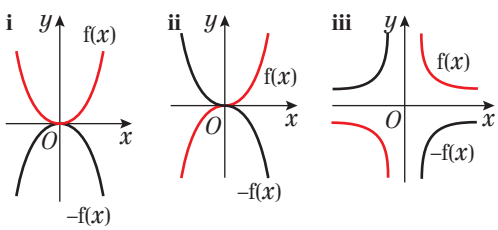

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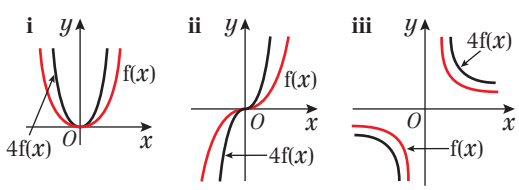
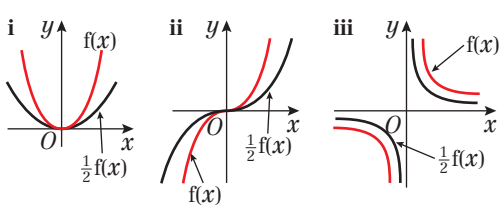
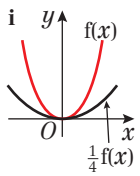


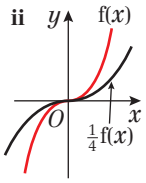
i



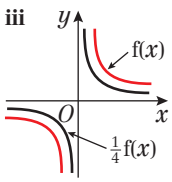
j **i**



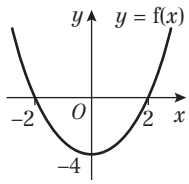
ii



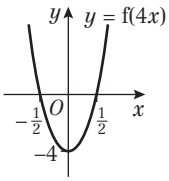
iii

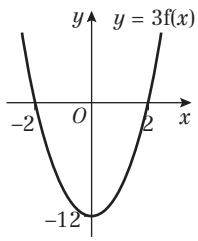


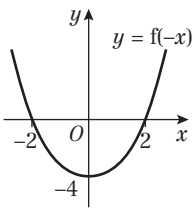
2 **a**

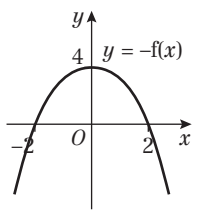


b

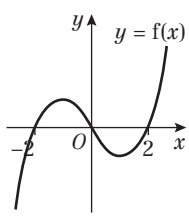








3 **a**



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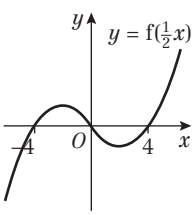
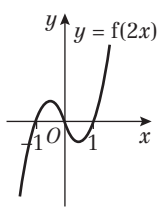
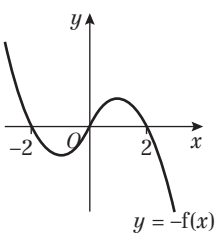
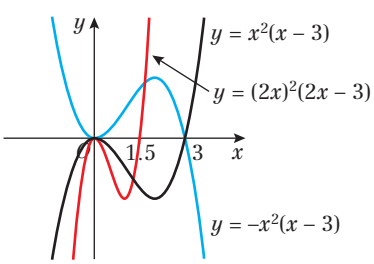






4



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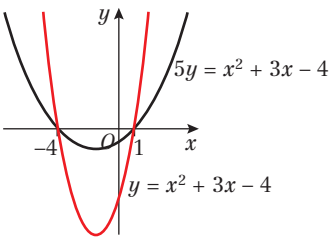

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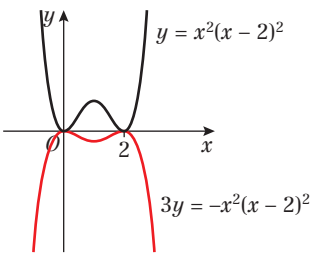
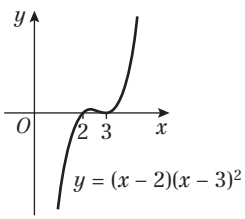


7 **a** *(1, −3)* **b** *(2, −12)*

8 *(−4, 8)*

9 **a**



b *2 dhe 3*

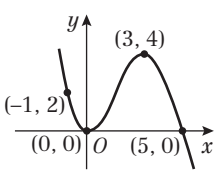
Sfidë

1 *(2, −2)*

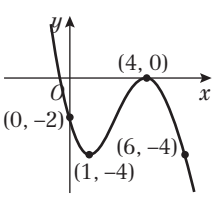
2 $\frac{1}{4}f(\frac{1}{2}x)$

Ushtrime 4G

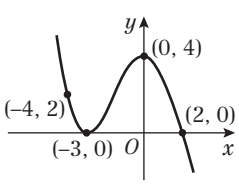
1 **a**



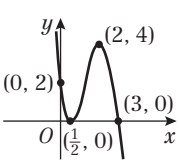
b



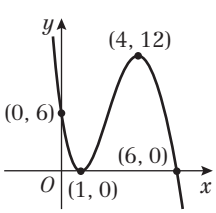
c



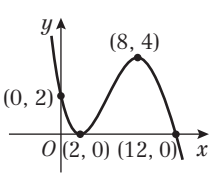
d



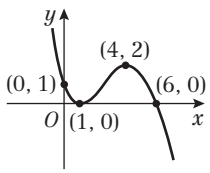
e



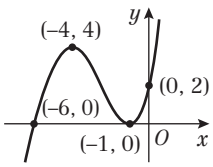
f



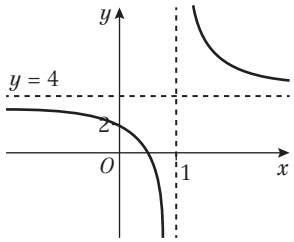
g



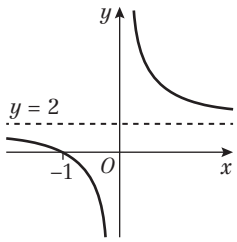
h



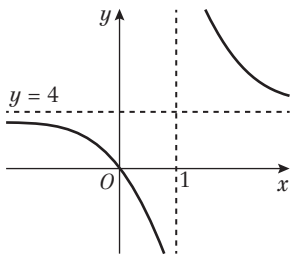
2 **a** *y = 4, x = 1, (0, 2)*



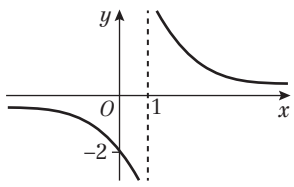
b *y = 2, x = 0, (−1, 0)*



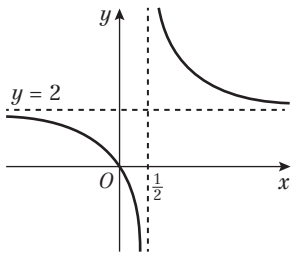
c *y = 4, x = 1, (0, 0)*



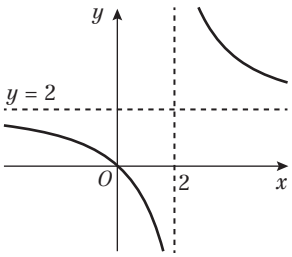
d *y = 0, x = 1, (0, −2)*



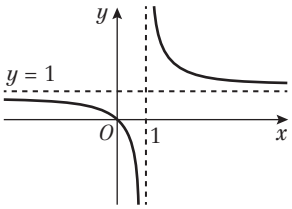
e *y = 2, x =* $\frac{1}{2}$ *(0, 0)*



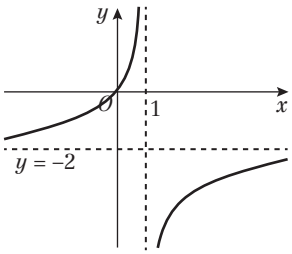
f *y = 2, x = 2, (0, 0)*



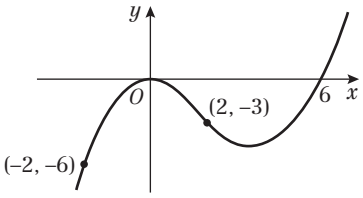
g *y = 1, x = 1, (0, 0)*



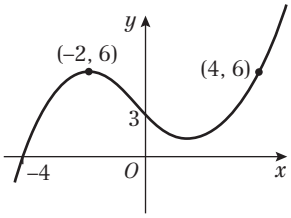
h *y = −2, x = 1, (0, 0)*



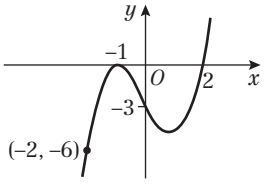
3 **a** *A(−2, −6), B(0, 0), C(2, −3), D(6, 0)*



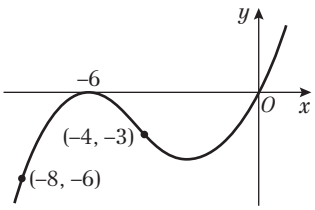
b *A(−4, 0), B(−2, 6), C(0, 3), D(4, 6)*



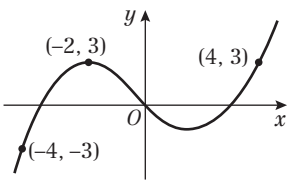
c *A(−2, −6), B(−1, 0), C(0, −3), D(2, 0)*



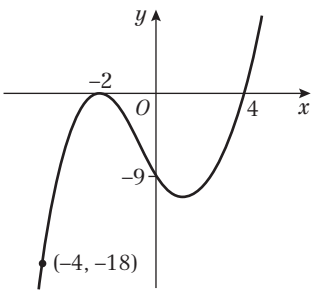
d *A(−8, −6), B(−6, 0), C(−4, −3), D(0, 0)*



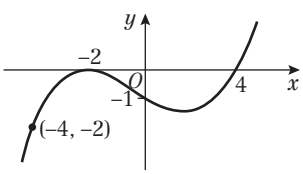
e *A(−4, −3), B(−2, 3), C(0, 0), D(4, 3)*



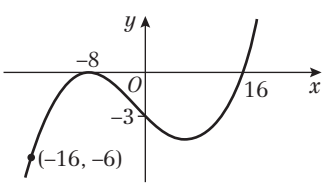
f *A(−4, −18), B(−2, 0), C(0, −9), D(4, 0)*



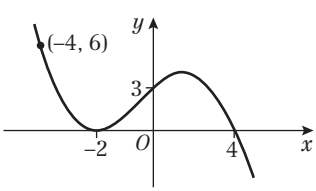
g *A(−4, −2), B(−2, 0), C(0, −1), D(4, 0)*



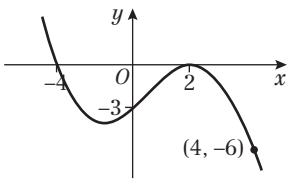
h *A(−16, −6), B(−8, 0), C(0, −3), D(16, 0)*



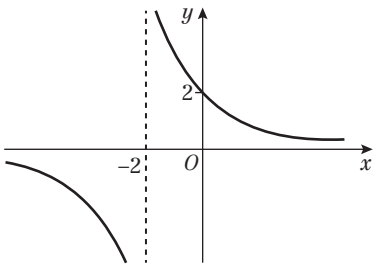
i *A(−4, 6), B(−2, 0), C(0, 3), D(4, 0)*



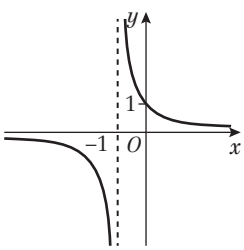
j *A(4, −6), B(2, 0), C(0, −3), D(−4, 0)*



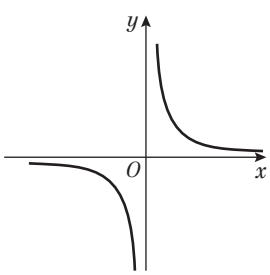
4 **a** **i** *x = −2, y = 0, (0, 2)*



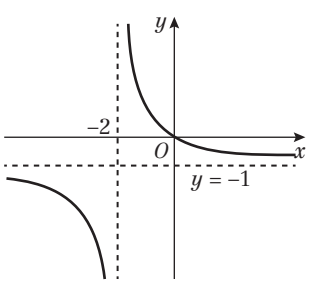
ii *x = −1, y = 0, (0, 1)*



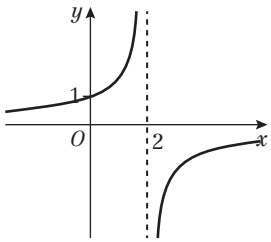
iii *x = 0, y = 0*



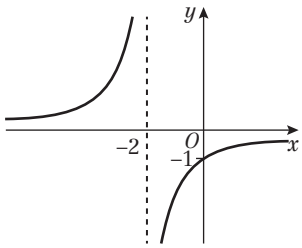
iv *x = −2, y = −1, (0, 0)*



v $x = 2, y = 0, (0, 1)$



vi $x = -2, y = 0, (0, -1)$

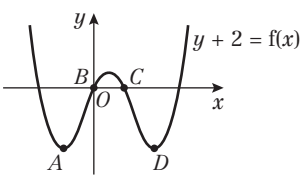


b $f(x) = \frac{2}{x+2}$

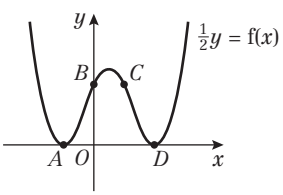
5 **a** $\frac{1}{2}$

b **i** $(6, 1)$ **ii** $(2, 3)$ **iii** $(2, -3.5)$

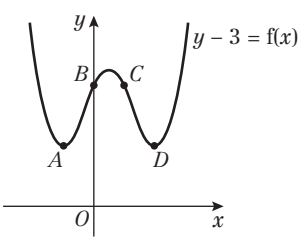
6 **a** $A(-1, -2)\ B(0, 0)\ C(1, 0)\ D(2, -2)$



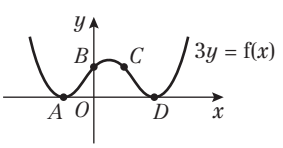
b $A(-1, 0)\ B(0, 4)\ C(1, 4)\ D(2, 0)$



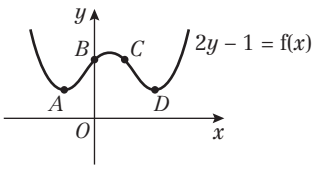
c $A(-1, 3)\ B(0, 5)\ C(1, 5)\ D(2, 3)$



d $A(-1, 0)\ B(0, \frac{2}{3})\ C(1, \frac{2}{3})\ D(2, 0)$



e $A(-1, 0.5)\ B(0, 1.5)\ C(1, 1.5)\ D(2, 0.5)$



KAPITULLI 5

Kontrolli i njohurive të mëparshme

1 **a** $(-2, -1)$ **b** $\left(\frac{9}{19}, \frac{26}{19}\right)$ **c** $(7, 3)$

2 **a** $4\sqrt{5}$ **b** $10\sqrt{2}$ **c** $5\sqrt{5}$

3 **a** $y = 5 - 2x$ **b** $y = \frac{2}{5}x - \frac{9}{5}$ **c** $y = \frac{3}{7}x + \frac{12}{7}$

Ushtrime 5A

1 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $-\frac{3}{5}$ **d** 2
e -1 **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** 8
i $\frac{2}{3}$ **j** -4 **k** $-\frac{1}{3}$ **l** $-\frac{1}{2}$
m 1 **n** $\frac{q^2 - p^2}{q - p} = q + p$

2 7
3 12
4 $4\frac{1}{3}$
5 $2\frac{1}{4}$
6 $\frac{1}{4}$
7 26
8 -5
9 *Koeficienti këndor i tangjentes AB = Koeficienti këndor i BC = 0.5; pika B është e përbashkët.*
10 *Koeficienti këndor i tangjentes AB = Koeficienti këndor i BC = −0.5; pika B është e përbashkët.*

Ushtrime 5B

1 **a** -2 **b** -1 **c** 3 **d** $\frac{1}{3}$
e $-\frac{2}{3}$ **f** $\frac{5}{4}$ **g** $\frac{1}{2}$ **h** 2
i $\frac{1}{2}$ **j** $\frac{1}{2}$ **k** -2 **l** $-\frac{3}{2}$

2 **a** 4 **b** -5 **c** $-\frac{2}{3}$ **d** 0
e $\frac{7}{5}$ **f** 2 **g** 2 **h** -2
i 9 **j** -3 **k** $\frac{3}{2}$ **l** $-\frac{1}{2}$

3 **a** $4x - y + 3 = 0$ **b** $3x - y - 2 = 0$
c $6x + y - 7 = 0$ **d** $4x - 5y - 30 = 0$
e $5x - 3y + 6 = 0$ **f** $7x - 3y = 0$
g $14x - 7y - 4 = 0$ **h** $27x + 9y - 2 = 0$
i $18x + 3y + 2 = 0$ **j** $2x + 6y - 3 = 0$
k $4x - 6y + 5 = 0$ **l** $6x - 10y + 5 = 0$

4 $(3, 0)$
5 $(0, 0)$
6 $(0, 5), (-4, 0)$
7 **a** $\frac{1}{3}$ **b** $x - 3y + 15 = 0$
8 **a** $-\frac{2}{5}$ **b** $2x + 5y - 10 = 0$
9 $ax + by + c = 0$
$by = -ax - c$
$y = \left(-\frac{a}{b}\right)x - \left(\frac{c}{b}\right)$
10 $a = 6, c = 10$
11 $P(3,0)$

12 **a** -16 **b** -27

Sfidë

Koeficienti këndor i tangjentes $= -\frac{a}{b}$; *pikëprerja me boshtin* $y = a$.
Pra $y = -\frac{a}{b}x + a$

Rishkruaj që të kesh $ax + by - ab = 0$

Ushtrime 5C

1 **a** $y = 2x + 1$ **b** $y = 3x + 7$ **c** $y = -x - 3$
d $y = -4x - 11$ **e** $y = \frac{1}{2}x + 12$ **f** $y = -\frac{2}{3}x - 5$
g $y = 2x$ **h** $y = -\frac{1}{2}x + 2b$

2 **a** $y = 4x - 4$ **b** $y = x + 2$ **c** $y = 2x + 4$
d $y = 4x - 23$ **e** $y = x - 4$ **f** $y = \frac{1}{2}x + 1$
g $y = -4x - 9$ **h** $y = -8x - 33$ **i** $y = \frac{6}{5}x$
j $y = \frac{2}{7}x + \frac{5}{14}$

3 $5x + y - 37 = 0$

4 $y = x + 2$, $y = -\frac{1}{6}x - \frac{1}{3}$, $y = -6x + 23$

5 $a = 3$, $c = -27$

6 $a = -4$, $b = 8$

Sfidë

a $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

b $y - y_1 = \frac{(y_2 - y_1)}{(x^2 - y_1)}(x - x_1)$

$\frac{(y - y_1)}{(y_2 - y_1)} = \frac{(x - x_1)}{(x_2 - x_1)}$

c $y = \frac{3}{7}x + \frac{52}{7}$

Ushtrime 5D

1 $y = 3x - 6$ **2** $y = 2x + 8$
3 $2x - 3y + 24 = 0$ **4** $-\frac{1}{5}$
5 $(-3, 0)$ **6** $(0, 1)$
7 $(0, 3\frac{1}{2})$ **8** $y = \frac{2}{5}x + 3$
9 $2x + 3y - 12 = 0$ **10** $\frac{8}{5}$
11 $y = \frac{4}{3}x - 4$ **12** $6x + 15y - 10 = 0$
13 $y = -\frac{4}{5}x + 4$ **14** $x - y + 5 = 0$
15 $y = -\frac{3}{8}x + \frac{1}{2}$ **16** $y = 4x + 13$

Ushtrime 5E

1 **a** *Paralel* **b** *jo paralel* **c** *jo paralel*

2 r: $y = \frac{4}{5}x + 3.2$, s: $y = \frac{4}{5}x - 7$
Koeficientet këndore janë të barabartë, rrjedhimisht drejtëzat janë paralele.

3 *Koeficienti këndor i AB* $= \frac{3}{5}$, *Koeficienti këndor i BC* $= -\frac{7}{2}$, *Koeficienti këndor i CD* $= \frac{3}{5}$, *Koeficienti këndor i AD* $= \frac{10}{3}$. *Katërkëndëshi ka dy brinjë paralele, pra ai është një trapez.*

4 $y = 5x + 3$

5 $2x + 5y + 20 = 0$

6 $y = -\frac{1}{2}x + 7$

7 $y = \frac{2}{3}x$

8 $4x - y + 15 = 0$

Ushtrime 5F

1 **a** *Perpendikular* **b** *Paralel*
c *Asnjëra* **d** *Perpendikular*
e *Perpendikular* **f** *Paralel*
g *Paralel* **h** *Perpendikular*
i *Perpendikular* **j** *Paralel*
k *Asnjëra* **l** *Perpendikular*

2 $y = -\frac{1}{6}x + 1$

3 $y = \frac{8}{3}x - 8$

4 $y = -\frac{1}{3}x$

5 $y = -\frac{1}{3}x + \frac{13}{3}$

6 $y = -\frac{3}{2}x + \frac{17}{2}$

7 $3x + 2y - 5 = 0$

8 $7x - 4y + 2 = 0$

9 *l e ka koeficientin këndor* $-\frac{1}{3}$ *dhe n e ka koeficientin këndor 3. Koeficientet këndore janë të anasjelltë të njëri-tjetrit dhe me shenjë të kundërt. Atëherë drejtëzat janë perpendikulare.*

10 *AB:* $y = -\frac{1}{2}x + 4\frac{1}{2}$, *CD:* $y = -\frac{1}{2}x - \frac{1}{2}$, *AD:* $y = 2x + 7$, *BC:* $y = 2x - 13$. *Dy çifte brinjësh paralele dhe drejtëzat me koeficient këndor 2 dhe* $-\frac{1}{2}$ *janë perpendikulare, pra ABCD është një drejtkëndësh.*

11 **a** $A(\frac{7}{5}, 0)$ **b** $55x - 25y - 77 = 0$

12 $-\frac{9}{4}$

Ushtrime 5G

1 **a** 10 **b** 13 **c** 5 **d** $\sqrt{5}$
e $\sqrt{106}$ **f** $\sqrt{113}$

2 *Distanca midis A dhe B* $= \sqrt{50}$ *dhe distanca midis B dhe C* $= \sqrt{50}$ *pra drejtëzat janë kongruente.*

3 *Distanca midis P dhe Q* $= \sqrt{74}$ *dhe distanca midis Q dhe R* $= \sqrt{73}$ *pra drejtëzat nuk janë kongruente.*

4 $x = -8$ *ose* $x = 6$

5 $y = -2$ *ose* $y = 16$

6 **a** *Të dy drejtëzat e kanë koeficientin këndor2.*
b $y = -\frac{1}{2}x + \frac{23}{2}$ *ose* $x + 2y - 23 = 0$
c $(\frac{29}{5}, \frac{43}{5})$
d $\frac{7\sqrt{5}}{5}$

7 $P(-\frac{3}{5}, \frac{29}{5})$ *ose* $P(3, -5)$

8 **a** $AB = \sqrt{178}$, $BC = 3$ *dhe* $AC = \sqrt{205}$. *Të gjitha brinjët kanë gjatësi të ndryshme, rrjedhimisht trekëndëshi është brinjëndryshëm.*
b $\frac{39}{2}$ *ose* 19.5

9 **a** $A(2, 11)$
b $B(\frac{41}{4}, 0)$
c $\frac{451}{8}$

10 **a** $(\frac{5}{2}, 0)$ **b** $(-5, 0)$
c $(-10, -10)$ **d** $\frac{75}{2}$

11 **a** $y = \frac{1}{2}x - \frac{9}{2}$ **b** $y = -2x + 8$
c $T(0, 8)$ **d** $RS = 2\sqrt{5}$ *dhe* $TR = 5\sqrt{5}$
e 25

12 **a** $x + 4y - 52 = 0$ **b** $A(0,13)$
c $B(4, 12)$ **d** 26

KAPITULLI 6

Kontrolli i njohurive të mëparshme

1 **a** $(x + 5)^2 + 3$ **b** $(x - 3)^2 - 8$
c $(x - 6)^2 - 36$ **d** $(x + \frac{7}{2})^2 - \frac{49}{4}$

2 **a** $y = \frac{9}{4}x - 6$ **b** $y = -\frac{1}{2}x - \frac{3}{2}$
c $y = \frac{4}{3}x + \frac{10}{3}$

3 **a** $b^2 - 4ac = -7$ Nuk ka zgjidhje reale
b $b^2 - 4ac = 89$ Dy zgjidhje reale
c $b^2 - 4ac = 0$ Një zgjidhje reale

4 $y = -\frac{5}{6}x - \frac{3}{2}$

Ushtrime 6A

1 **a** $(5, 5)$ **b** $(6, 4)$ **c** $(-1, 4)$ **d** $(0, 0)$
e $(2, 1)$ **f** $(-8, \frac{3}{2})$ **g** $(4a, 0)$ **h** $(-\frac{u}{2}, -v)$
i $(2a, a - b)$ **j** $(3\sqrt{2}, 4)$ **k** $(2\sqrt{2}, \sqrt{2} + 3\sqrt{3})$

2 $a = 10, b = 1$

3 $(\frac{3}{2}, 7)$

4 $(\frac{3a}{5}, \frac{b}{4})$

5 **a** $(\frac{3}{2}, 3)$ ose $(1.5, 3)$ **b** $y = 2x, 3 = 2 \times 1.5$

6 **a** $(\frac{1}{6}, \frac{5}{3})$ **b** $\frac{2}{3}$

7 Qendra është $(3, -\frac{7}{2})$. $3 - 2(-\frac{7}{2}) - 10 = 0$

8 $(10, 5)$

9 $(-7a, 17a)$

10 $p = 8, q = 7$

11 $a = -2, b = 4$

Sfidë

a $p = 9, q = -1$
b $y = -x + 13$
c $AC: y = -x + 8$. Drejtëzat kanë të njëjtin koeficient këndor, pra ato janë paralele.

Ushtrime 6B

1 **a** $y = 2x + 3$ **b** $y = -\frac{1}{3}x + \frac{47}{3}$ **c** $y = \frac{5}{2}x - 25$
d $y = 3$ **e** $y = -\frac{3}{4}x + \frac{37}{8}$ **f** $x = 9$

2 $y = -x + 7$

3 $2x - y - 8 = 0$

4 **a** $y = -\frac{5}{3}x - \frac{13}{3}$ **b** $y = 3x - 8$ **c** $(\frac{11}{14}, -\frac{79}{14})$

5 $q = -\frac{5}{4}, b = -\frac{189}{8}$

Sfidë

a $PR: y = -\frac{5}{2}x + \frac{9}{4}$
$PQ: y = -\frac{1}{4}x + \frac{33}{8}$
$RQ: y = 2x + 6$

b $(-\frac{5}{6}, \frac{13}{3})$

Ushtrime 6C

1 **a** $(x - 3)^2 + (y - 2)^2 = 16$
b $(x + 4)^2 + (y - 5)^2 = 36$
c $(x - 5)^2 + (y + 6)^2 = 12$
d $(x - 2a)^2 + (y - 7a)^2 = 25a^2$
e $(x + 2\sqrt{2})^2 + (y + 3\sqrt{2})^2 = 1$

2 **a** $(-5, 4), 9$ **b** $(7, 1), 4$
c $(-4, 0), 5$ **d** $(-4a, -a), 12a$
e $(3\sqrt{5}, -\sqrt{5}), 3\sqrt{3}$

3 **a** $(4 - 2)^2 + (8 - 5)^2 = 4 + 9 = 13$
b $(0 + 7)^2 + (-2 - 2)^2 = 49 + 16 = 65$
c $7^2 + (-24)^2 = 49 + 576 = 625 = 25^2$
d $(6a - 2a)^2 + (-3a + 5a)^2 = 16a^2 + 4a^2 = 20a^2$
e $(\sqrt{5} - 3\sqrt{5})^2 + (-\sqrt{5} - \sqrt{5})^2 = (-2\sqrt{5})^2 + (-2\sqrt{5})^2$
$= 20 + 20 = 40 = (2\sqrt{10})^2$

4 $(x - 8)^2 + (y - 1)^2 = 25$

5 $(x - \frac{3}{2})^2 + (y - 4)^2 = \frac{65}{4}$

6 $\sqrt{5}$

7 **a** $r = 2$
b Distanca $PQ = PR = RQ = 2\sqrt{3}$, tre gjatësi brinjësh të barabarta, rrjedhimisht trekëndëshi është barabrinjës.

8 **a** $(x - 2)^2 + y^2 = 15$
b Qendra $(2, 0)$ dhe rrezja $= \sqrt{15}$

9 **a** $(x - 5)^2 + (y + 2)^2 = 49$
b Qendra $(5, -2)$ dhe rrezja $= 7$

10 **a** Qendra $(1, -4)$, dhe rrezja 5
b Qendra $(-6, 2)$, dhe rrezja 7
c Qendra $(11, 3)$, dhe rrezja $3\sqrt{10}$
d 10 Qendra $(-2.5, 1.5)$, dhe rrezja $\frac{5\sqrt{2}}{2}$
e Qendra $(2, -2)$, dhe rrezja $\sqrt{6.5}$

11 **a** Qendra $(-6, -1)$
b $k > -37$

12 $Q(-13, 28)$

13 $k = -2$ dhe $k = 8$

Sfidë

1 $k = 3, (x - 3)^2 + (y - 2)^2 = 50$
$k = 5, (x - 5)^2 + (y - 2)^2 = 50$

2 $(x + f)^2 - f^2 + (y + g)^2 - g^2 + c = 0$
So $(x + f)^2 + (y + g)^2 = f^2 + g^2 - c$
Rrethi me qendër $(-f, -g)$ dhe rreze $\sqrt{f^2 + g^2 - c}$.

Ushtrime 6D

1 $(7, 0), (-5, 0)$
2 $(0, 2), (0, -8)$
3 $(6, 10), (-2, 2)$
4 $(4, -9), (-7, 2)$
5 $2x^2 - 24x + 79 = 0$ nuk ka zgjidhje reale, rrjedhimisht drejtëzat nuk presin rrethin.
6 **a** $b^2 - 4ac = 64 - 4 \times 1 \times 16 = 0$. Rrjedhimisht ka vetëm një pikë prerje.
b $(4, 7)$
7 **a** $(0, -2), (4, 6)$ **b** pika e mesit e AB është $(2, 2)$
8 **a** 13 **b** $p = 1$ ose 5
9 **a** $A(5, 0)$ dhe $B(-3, -8)$ (ose anasjelltas)
b $y = -x - 3$
c $(4, -7)$ është zgjidhje e $y = -x - 3$.
d 20
10 **a** Zëvendëso $y = kx$ dhe gjen
$(k^2 + 1)x^2 - (12k + 10)x + 57 = 0$
$b^2 - 4ac > 0, -84k^2 + 240k - 128 > 0,$
$21k^2 - 60k + 32 < 0$
b $0.71 < k < 2.15$
Përgjigja e saktë është $\frac{10}{7} - \frac{2\sqrt{57}}{21} < k < \frac{10}{7} + \frac{2\sqrt{57}}{21}$
11 $k < \frac{8}{17}$
12 $k = -20 \pm 2\sqrt{105}$

Ushtrime 6E

1 **a** $3\sqrt{10}$
b Koeficienti këndor i rrezes = 3, koeficienti këndor i drejtëzës $= -\frac{1}{3}$, koeficientet këndore janë të anasjelltë të njëri-tjetrit dhe me shenjë të kundërt dhe rrjedhimisht perpendikulare.
2 **a** $(x - 4)^2 + (y - 6)^2 = 73$ **b** $3x + 8y + 13 = 0$
3 **a** $y = -2x - 1$
b Qendra e rrethit $(1, -3)$ kënaq $y = -2x - 1$.
4 **a** $y = \frac{1}{2}x - 3$
b Qendra e rrethit $(2, -2)$ kënaq $y = \frac{1}{2}x - 3$
5 **a** $(-7, -6)$ kënaq $x^2 + 18x + y^2 - 2y + 29 = 0$
b $y = \frac{2}{7}x - 4$ **c** $R(0, -4)$ **d** $\frac{53}{2}$
6 **a** $(0, -17), (17, 0)$
b 144.5
7 $y = 2x + 27$ dhe $y = 2x - 13$
8 **a** $p = 4, p = -6$
b $(3, 4)$ dhe $(3, -6)$
9 **a** $(x - 11)^2 + (y + 5)^2 = 100$
b $y = \frac{3}{4}x - \frac{3}{4}$
c $A(8 - 4\sqrt{3}, -1 - 3\sqrt{3})$ dhe $B(8 + 4\sqrt{3}, -1 + 3\sqrt{3})$
d $10\sqrt{3}$
10 **a** $y = 4x - 22$
b $a = 5$
c $(x - 5)^2 + (y + 2)^2 = 34$
d $A(5 + \sqrt{2}, -2 + 4\sqrt{2})$ dhe $B(5 - \sqrt{2}, -2 - 4\sqrt{2})$
11 **a** $P(-2, 5)$ dhe $Q(4, 7)$
b $y = 2x + 9$ dhe $y = -\frac{1}{2}x + 9$
c $y = -3x + 9$
d $(0, 9)$

Sfidë

1 $y = \frac{1}{2}x - 2$
2 **a** $\angle CPR = \angle CQR = 90°$ (Këndi midis tangjentes dhe rrezes)
$CP = CQ = \sqrt{10}$ (Rrezja e rrethit)
$CR = \sqrt{(6 - 2)^2 + (-1 - 1)^2} = \sqrt{20}$
(Pra nga teorema e Pitagorës,
$PR = QR = \sqrt{20 - 10} = \sqrt{10}$
4 brinjë të barabarta dhe dy kënde të kundërt të drejtë pra CPRQ është një katror.
b $y = \frac{1}{3}x - 3$ dhe $y = -3x + 17$

Ushtrime 6F

1 **a** $WV^2 = WU^2 + UV^2$
b $(2, 3)$
c $(x - 2)^2 + (y - 3)^2 = 41$
2 **a** $AC^2 = AB^2 + BC^2$
b $(x - 5)^2 + (y - 2)^2 = 25$
c 15
3 **a** **i** $y = \frac{3}{2}x + \frac{21}{2}$ **ii** $y = -\frac{2}{3}x + 4$
b $(-3, 6)$
c $(x + 3)^2 + (y - 6)^2 = 169$
4 **a** **i** $y = \frac{1}{3}x + \frac{10}{3}$ **ii** $x = -1$
b $(x + 1)^2 + (y - 3)^2 = 125$
5 $(x - 3)^2 + (y + 4)^2 = 50$
6 **a** $AB^2 + BC^2 = AC^2$
$AB^2 = 400, BC^2 = 100, AC^2 = 500$
b $(x + 2)^2 + (y - 5)^2 = 125$
c $D(8, 0)$ kënaq ekuacionin e rrethit.
7 **a** $AB = BC = CD = DA = \sqrt{50}$
b 50
c $(3,6)$
8 **a** $DE^2 = b^2 + 6b + 13$
$EF^2 = b^2 + 10b + 169$
$DF^2 = 200$
So $b^2 + 6b + 13 + b^2 + 10b + 169 = 200$
$(b + 9)(b - 1) = 0$; as $b > 0$, $b = 1$
b $(x + 5)^2 + (y + 4)^2 = 50$
9 **a** Qendra $(-1, 12)$ dhe rrezja = 13
b Me anë të formulës së distancës gjej AB = 26. Kjo është dy herë sa rrezja, pra AB është një diametër. Ka edhe metoda të tjera të mundshme.
c $C(-6, 0)$

KAPITULLI 7

Kontrolli i njohurive të mëparshme

1 **a** $15x^7$ **b** $\frac{x}{3y}$
2 **a** $(x - 6)(x + 4)$ **b** $(3x - 5)(x - 4)$
3 **a** 8567 **b** 1652
4 **a** $y = 1 - 3x$ **b** $y = \frac{1}{2}x - 7$
5 **a** $(x - 1)^2 - 21$ **b** $2(x + 1)^2 + 13$

Ushtrime 7A

1 **a** $4x^3 + 5x - 7$ **b** $2x^4 + 9x^2 + x$
c $-x^3 + 4x + \frac{6}{x}$ **d** $7x^4 - x^2 - \frac{4}{x}$
e $4x^3 - 2x^2 + 3$ **f** $3x - 4x^2 - 1$

g $\frac{7x^2}{5}-\frac{x^3}{5}-\frac{2}{5x}$ **h** $2x-3x^3+1$

i $\frac{x^7}{2}-\frac{9x^3}{2}+2x^2-\frac{3}{x}$ **j** $3x^8+2x^5-\frac{4x^3}{3}+\frac{2}{3x}$

2 **a** $x+3$ **b** $x+4$ **c** $x+3$
d $x+7$ **e** $x+5$ **f** $x+4$
g $\frac{x-4}{x-3}$ **h** $\frac{x+2}{x+4}$ **i** $\frac{x+4}{x-6}$
j $\frac{2x+3}{x-5}$ **k** $\frac{2x-3}{x+1}$ **l** $\frac{x-2}{x+2}$
m $\frac{2x+1}{x-2}$ **n** $\frac{x+4}{3x+1}$ **o** $\frac{2x+1}{2x-3}$

3 $a = 1, b = 4, c = -2$

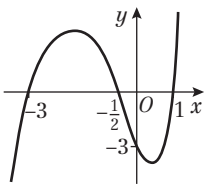
Ushtrime 7B

1 **a** $(x+1)(x^2+5x+3)$ **b** $(x+4)(x^2+6x+1)$
c $(x+2)(x^2-3x+7)$ **d** $(x-3)(x^2+4x+5)$
e $(x-5)(x^2-3x-2)$ **f** $(x-7)(x^2+2x+8)$
2 **a** $(x+4)(6x^2+3x+2)$ **b** $(x+2)(4x^2+x-5)$
c $(x+3)(2x^2-2x-3)$ **d** $(x-6)(2x^2-3x-4)$
e $(x+6)(-5x^2+3x+5)$ **f** $(x-2)(-4x^2+x-1)$
3 **a** x^3+3x^2-4x+1 **b** $4x^3+2x^2-3x-5$
c $-3x^3+3x^2-4x-7$ **d** $-5x^4+2x^3+4x^2-3x+7$
4 **a** x^3+2x^2-5x+4 **b** x^3-x^2+3x-1
c $2x^3+5x+2$ **d** $3x^4+2x^3-5x^2+3x+6$
e $2x^4-2x^3+3x^2+4x-7$ **f** $4x^4-3x^3-2x^2+6x-5$
g $5x^3+12x^2-6x-2$ **h** $3x^4+5x^3+6$
5 **a** x^2-2x+5 **b** $2x^2-6x+1$
c $-3x^2-12x+2$
6 **a** $x^2+4x+12$ **b** $2x^2-x+5$
c $-3x^2+5x+10$
7 *Pjesëto* $x^3+2x^2-5x-10$ *me* $(x+2)$ *dhe gjej* (x^2-5). *Pra* $x^3+2x^2-5x-10=(x+2)(x^2-5)$.
8 **a** -8 **b** -7 **c** -12
9 $f(1) = 3-2+4 = 5$
10 $f(-1) = 3+8+10+3-25 = -1$
11 $(x+4)(5x^2-20x+7)$
12 $3x^2+6x+4$
13 x^2+x+1
14 x^3-2x^2+4x-8
15 14
16 **a** -200 **b** $(x+2)(x-7)(3x+1)$
17 **a** **i** 30 **ii** 0 **b** $x=-3, x=-4, x=1$
18 **a** $a = 1, b = 2, c = -3$
b $f(x) = (2x-1)(x+3)(x-1)$
c $x = 0.5, x = -3, x = 1$
19 **a** $a = 3, b = 2, c = 1$
b *Shprehja kuadratike nuk ka zgjidhje reale, pra vetëm* $\frac{1}{4}$ *është një zgjidhje.*

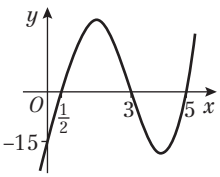
Ushtrime 7C

1 **a** $f(1) = 0$ **b** $f(-3) = 0$ **c** $f(4) = 0$
2 $(x-1)(x+3)(x+4)$
3 $(x+1)(x+7)(x-5)$
4 $(x-5)(x-4)(x+2)$
5 $(x-2)(2x-1)(x+4)$
6 **a** $(x+1)(x-5)(x-6)$ **b** $(x-2)(x+1)(x+2)$
c $(x-5)(x+3)(x-2)$

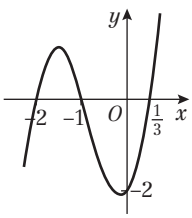
7 **a** **i** $(x-1)(x+3)(2x+1)$ **ii**



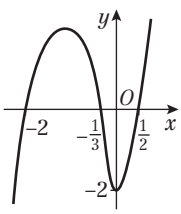
b **i** $(x-3)(x-5)(2x-1)$ **ii**



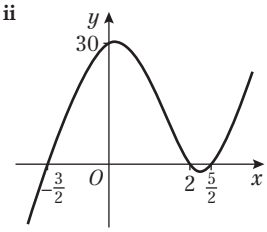
c **i** $(x+1)(x+2)(3x-1)$ **ii**



d **i** $(x+2)(2x-1)(3x+1)$ **ii**



e **i** $(x-2)(2x-5)(2x+3)$ **ii**



8 2
9 -16
10 $p = 3, q = 7$
11 $c = 2, d = 3$
12 $g = 3, h = -7$
13 **a** $f(4) = 0$
b $f(x) = (x-4)(3x^2+6)$
For $3x^2+6=0$, $b^2-4ac=-72$ *pra there are no real roots. Therefore, 4 është the only real root of* $f(x) = 0$.
14 **a** $f(-2) = 0$ **b** $(x+2)(2x+1)(2x-3)$
c $x=-2, x=-\frac{1}{2}$ *dhe* $x=1\frac{1}{2}$
15 **a** $f(2) = 0$ **b** $x=0, x=2, x=-\frac{1}{3}$ *dhe* $x=\frac{1}{3}$

Sfidë

a $f(1) = 2-5-42-9+54 = 0$
$f(-3) = 162+135-378+27+54 = 0$
b $2x^4-5x^3-42x^2-9x+54$
$=(x-1)(x+3)(x-6)(2x+3)$
$x = 1\ x = -3, x = 6, x = -1.5$

Ushtrime 7D

1 $n^2 - n = n(n-1)$
Në qoftë se n është çift, atëherë, $n - 1$ është tek dhe çift × tek = çift.
Në qoftë se n është tek atëherë, $n - 1$ është çift dhe tek × çift = çift

2 $\frac{x}{(1+\sqrt{2})} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})} = \frac{x(1-\sqrt{2})}{(1-2)} = \frac{x - x\sqrt{2}}{-1} = x\sqrt{2} - x$

3 $(x + \sqrt{y})(x - \sqrt{y}) = x^2 - x\sqrt{y} + x\sqrt{y} - y = x^2 - y$

4 $(2x-1)(x+6)(x-5) = (2x-1)(x^2 + x - 30)$
$= 2x^3 + x^2 - 61x + 30$

5 Ana e majtë $= x^2 + bx$, duke përdorur plotësimin e katrorit,
$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

6 $x^2 + 2bx + c = 0$, duke përdorur plotësimin e katrorit,
$(x+b)^2 + c - b^2 = 0$
$(x+b)^2 = b^2 - c$
$x + b = \pm\sqrt{b^2 - c}$
$x = -b \pm \sqrt{b^2 - c}$

7 $\left(x - \frac{2}{x}\right)^3 = \left(x - \frac{2}{x}\right)\left(x^2 - 4 + \frac{4}{x^2}\right) = x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}$

8 $\left(x^3 - \frac{1}{x}\right)\left(x^{\frac{3}{2}} + x^{\frac{-3}{2}}\right) = x^{\frac{9}{2}} + x^{\frac{1}{2}} - x^{\frac{1}{2}} - x^{\frac{-7}{2}} = x^{\frac{9}{2}} - x^{\frac{-7}{2}}$
$= x^{\frac{1}{2}}\left(x^4 - \frac{1}{x^4}\right)$

9 $3n^2 - 4n + 10 = 3\left[n^2 - \frac{4}{3}n + \frac{10}{3}\right] = 3\left[\left(n - \frac{2}{3}\right)^2 + \frac{10}{3} - \frac{4}{9}\right]$
$= 3\left(n - \frac{2}{3}\right)^2 + \frac{26}{3}$
Vlera minimale është $\frac{26}{3}$ pra $3n^2 - 4n + 10$ është gjithnjë pozitive.

10 $-n^2 - 2n - 3 = -[n^2 + 2n + 3] = -[(n+1)^2 + 3 - 1]$
$= -(n+1)^2 - 2$
Vlera maksimale është -2 pra $-n^2 - 2n - 3$ është gjithnjë negative.

11 $x^2 + 8x + 20 = (x+4)^2 + 4$
Vlera minimale është 4 pra $x^2 + 8x + 20$ është përherë më e madhe ose e barabartë me 4. 4.

12 $kx^2 + 5kx + 3 = 0$, $b^2 - 4ac < 0$, $25k^2 - 12k < 0$,
$k(25k - 12) < 0$, $0 < k < \frac{12}{25}$.
Kur $k = 0$ nuk ka rrënjë reale, pra $0 \le k < \frac{12}{25}$

13 $px^2 - 5x - 6 = 0$, $b^2 - 4ac > 0$, $25 + 24p > 0$, $p > -\frac{25}{24}$

14 Koeficienti këndor $AB = -\frac{1}{2}$, koeficienti këndor $BC = 2$,
Koeficienti këndor AB × koeficienti këndor $BC = -\frac{1}{2} \times 2 = -1$,
pra AB dhe BC janë paralele.

15 Koeficienti këndor $AB = 3$, koeficienti këndor $BC = \frac{1}{4}$, koeficienti këndor $CD = 3$, koeficienti këndor $AD = \frac{1}{4}$
Koeficienti këndor AB = koeficienti këndor CD pra AB dhe CD janë paralele.
Koeficienti këndor BC = koeficienti këndor AD pra BC dhe AD janë paralele.

16 Koeficienti këndor $AB = \frac{1}{3}$, koeficienti këndor $BC = 3$, koeficienti këndor $CD = \frac{1}{3}$, koeficienti këndor $AD = 3$
Koeficienti këndor AB = koeficienti këndor CD pra AB dhe CD janë paralele.
Koeficienti këndor BC = koeficienti këndor AD pra BC dhe AD janë paralele.
Gjatësi $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$ dhe $AD = \sqrt{10}$,
pra të katër brinjët janë të barabarta..

17 Koeficienti këndor $AB = -3$, koeficienti këndor $BC = \frac{1}{3}$,
Koeficienti këndor AB × koeficienti këndor $BC = -3 \times \frac{1}{3} = -1$,
pra AB dhe BC janë paralele.
Gjatësia $AB = \sqrt{40}$, $BC = \sqrt{40}$, $AB = BC$

18 $(x-1)^2 + y^2 = k$, $y = ax$, $(x-1)^2 + a^2x^2 = k$,
$x^2(1 + a^2) - 2x + 1 - k = 0$
$b^2 - 4ac > 0$, $k > \frac{a^2}{1 + a^2}$.

19 $x = 2$. Ka vetëm një zgjidhje, pra drejtëza
$4y - 3x + 26 = 0$ vetëm sa e prek rrethin në një pikë, pra është tangent ndaj rrethit.

20 Syprina e katrorit $= (a+b)^2 = a^2 + 2ab + b^2$
Syprina e ngjyrosur $= 4\left(\frac{1}{2}ab\right)$
Syprina e katrorit të vogël: $a^2 + 2ab + b^2 - 2ab$
$= a^2 + b^2 = c^2$

Sfidë

1 Ekuacioni i rrethit është $(x-3)^2 + (y-5)^2 = 25$ dhe të katër pikat kënaqin këtë ekuacion.

2 $2k + 1 = 1 \times (2k+1) = ((k+1) - k)((k+1) + k) = (k+1)^2 - k^2$

Ushtrime 7E

1 3, 4, 5, 6, 7 dhe 8 nuk plotpjesëtohen me 10.

2 3, 5, 7, 11, 13, 17, 19, 23 janë numra të thjeshtë. 9, 15, 21, 25 janë prodhim i dy numrave të thjeshtë.

3 $1^2 + 2^2 = 5$, $2^2 + 3^2 = $ tek, $3^2 + 4^2 = $ tek, $4^2 + 5^2 = $ tek, $5^2 + 6^2 = $ tek, $6^2 + 7^2 = $ tek, $7^2 + 8^2 = 113$

4 $(3n)^3 = 27n^3 = 9n(3n^2)$ që është shumëfish i 9 9
$(3n+1)^3 = 27n^3 + 27n^2 + 9n + 1 = 9n(3n^2 + 3n + 1) + 1$ i cili është një më shumë se një shumëfish i 9
$(3n+2)^3 = 27n^3 + 54n^2 + 36n + 8 = 9n(3n^2 + 6n + 4) + 8$ i cili është një më shumë se një shumëfish i 9

5 **a** Për shembull, kur $n = 2$, $2^4 - 2 = 14$, 14 nuk plotpjesëtohet me 4.
b Çdo numër katror.
c Për shembull, kur $n = \frac{1}{2}$
d Për shembull, kur $n = 1$

6 **a** Duke supozuar se x dhe y janë pozitive.
b p.sh. $x = 0$, $y = 0$

7 $(x+5)^2 \ge 0$ për të gjitha vlerat x, dhe
$(x+5)^2 + 2x + 11 = (x+6)^2$, pra $(x+6)^2 \perp 2x + 11$

8 Në qoftë se $a^2 + 1 \perp 2a$ (a është pozitive, pra duke shumëzuar të dy anët me $a^2 - 2a + 1 \ge 0$, nuk ndryshon shenja e inekuacionit), atëherë dhe $(a-1)^2 \ge 0$, dhe ne e dimë se kjo nuk është e vërtetë.

9 **a** $(p+q)^2 = p^2 + 2pq + q^2 = (p-q)^2 + 4pq$
$(p-q)^2 \ge 0$ sepse është një katror, pra $(p+q)^2 \ge 4pq$
$p > 0$, $q > 0 \Rightarrow p + q > 0 \Rightarrow p + q \perp \sqrt{4pq}$
b p.sh. $p = q = -1$: $p + q = -2$, $\sqrt{4pq} = 2$

10 **a** Fillo duke supozuar se inekuacioni është i vërtetë: pra negative ≥ positiv
b p.sh. $x = y = -1$: $x + y = -2$, $\sqrt{x^2 + y^2} = \sqrt{2}$
c $(x+y)^2 = x^2 + 2xy + y^2 > x^2 + y^2$ sepse $x > 0$, $y > 0 \Rightarrow 2xy > 0$
Me qenë se $x + y > 0$, merr rrënjët katrore: $x + y \ge \sqrt{x^2 + y^2}$

Ushtrime 8A

1 **a** Rreshti i 4 **b** rreshti i 16
c rreshti i $(n+1)$ **d** rreshti i $(n+5)$

2 **a** $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
b $p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$
c $a^3 - 3a^2b + 3ab^2 - b^3$

d $x^3 + 12x^2 + 48x + 64$
e $16x^4 - 96x^3 + 216x^2 - 216x + 81$
f $a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32$
g $81x^4 - 432x^3 + 864x^2 - 768x + 256$
h $16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$

3 **a** 16 **b** -10 **c** 8 **d** 1280
e 160 **f** -2 **g** 40 **h** -96

4 $1 + 9x + 30x^2 + 44x^3 + 24x^4$

5 $8 + 12y + 6y^2 + y^3$, $8 + 12x - 6x^2 - 11x^3 + 3x^4 + 3x^5 - x^6$

6 ± 3

7 $\frac{5}{2}$, -1

8 $12p$

9 $500 + 25X + \frac{X^2}{2}$

Sfidë

$\frac{3}{4}$

Ushtrime 8B

1 **a** 24 **b** $362\,880$ **c** 720 **d** 210

2 **a** 6 **b** 15 **c** 20 **d** 5
e 45 **f** 126

3 **a** 5005 **b** 120 **c** $184\,756$ **d** 1140
e 2002 **f** 8568

4 $a = {}^4C_1$, $b = {}^5C_2$, $c = {}^6C_2$, $d = {}^6C_3$

5 330

6 **a** $120, 210$ **b** 960

7 **a** $286, 715$ **b** $57\,915$

8 *0.1762 me 4 shifra dhjetore. Megjithëse duket probabilitet i vogël, shanset janë më të mëdha që monedha të bjerë 10 herë kokë se sa çdo numër tjetër herësh rëniesh kokë.*

9 **a** ${}^nC_1 = \frac{n!}{1!(n-1)!}$

$$= \frac{1 \times 2 \times \ldots \times (n-2) \times (n-1) \times n}{1 \times 1 \times 2 \times \ldots \times (n-3) \times (n-2) \times (n-1)} = n$$

b ${}^nC_2 = \frac{n!}{2!(n-2)!}$

$$= \frac{1 \times 2 \times \ldots \times (n-2) \times (n-1) \times n}{1 \times 2 \times 1 \times 2 \times \ldots \times (n-3) \times (n-2)} = \frac{n(n-1)}{2}$$

10 $a = 37$

11 $p = 17$

Sfidë

a ${}^{10}C_3 = \frac{10!}{3!7!} = 120$ dhe ${}^{10}C_7 = \frac{10!}{7!3!} = 120$

b ${}^{14}C_5 = \frac{14!}{5!9!} = 2002$ dhe ${}^{14}C_9 = \frac{14!}{9!5!} = 2002$

c *Të dyja përgjigjet për pikën* **a** *janë të njëjta dhe të dyja përgjigjet për pikën* **b** *janë të njëjta.*

d ${}^nC_r = \frac{n!}{r!(n-r)!}$ dhe ${}^nC_{n-r} = \frac{n!}{(n-r)!r!}$ rrjedhimisht ${}^nC_r = {}^nC_{n-r}$

Ushtrime 8C

1 **a** $1 + 4x + 6x^2 + 4x^3 + x^4$
b $81 + 108x + 54x^2 + 12x^3 + x^4$
c $256 - 256x + 96x^2 - 16x^3 + x^4$
d $x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$
e $1 + 8x + 24x^2 + 32x^3 + 16x^4$
f $1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$

2 **a** $1 + 10x + 45x^2 + 120x^3$
b $1 - 10x + 40x^2 - 80x^3$
c $1 + 18x + 135x^2 + 540x^3$
d $256 - 1024x + 1792x^2 - 1792x^3$
e $1024 - 2560x + 2880x^2 - 1920x^3$
f $2187 - 5103x + 5103x^2 - 2835x^3$

3 **a** $64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3$
b $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3$
c $p^8 - 8p^7q + 28p^6q^2 - 56p^5q^3$
d $729x^6 - 1458x^5y + 1215x^4y^2 - 540x^3y^3$
e $x^8 + 16x^7y + 112x^6y^2 + 448x^5y^3$
f $512x^9 - 6912x^8y + 41\,472x^7y^2 - 145\,152x^6y^3$

4 **a** $1 + 8x + 28x^2 + 56x^3$
b $1 - 12x + 60x^2 - 160x^3$
c $1 + 5x + \frac{45}{4}x^2 + 15x^3$
d $1 - 15x + 90x^2 - 270x^3$
e $128 + 448x + 672x^2 + 560x^3$
f $27 - 54x + + 36x^2 - 8x^3$
g $64 - 576x + 2160x^2 - 4320x^3$
h $256 + 256x + 96x^2 + 16x^3$
i $128 + 2240x + 16\,800x^2 + 70\,000x^3$

5 $64 - 192x + 240x^2$

6 $243 - 810x + 1080x^2$

7 $x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}$

Sfidë

a $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
$(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$
$(a + b)^4 - (a - b)^4 = 8a^3b + 8ab^3 = 8ab(a^2 + b^2)$

b $82\,896 = 2^4 \times 3 \times 11 \times 157$

Ushtrime 8D

1 **a** 90 **b** 80 **c** -20
d 1080 **e** 120 **f** -4320
g 1140 **h** $-241\,920$ **i** -2.5
j 354.375 **k** -224 **l** 3.90625

2 $a = \pm\frac{1}{2}$

3 $b = -2$

4 $\cancel{1}$, $\frac{5 \pm \sqrt{105}}{8}$

5 **a** $p = 5$ **b** -10 **c** -80

6 **a** $5^{30} + 5^{29} \times 30px + 5^{28} \times 435p^2x^2$
b $p = 10$

7 **a** $1 + 10qx + 45q^2x^2 + 120q^3x^3$
b $q = \pm 3$

8 **a** $1 + 11px + 55p^2x^2$
b $p = 7$, $q = 2695$

9 **a** $1 + 15px + 105p^2x^2$
b $p = -\frac{5}{7}$ $q = 10\frac{5}{7}$

10 $\frac{q}{p} = 2.1$

Sfidë

a $314\,928$ **b** $43\,750$

KAPITULLI 9

Kontrolli i njohurive të mëparshme

1 **a** $3.10\ cm$ **b** $9.05\ cm$

2 **a** $25.8°$ **b** $77.2°$

3 **a** *grafiku i* $x^2 + 3x$ **b** *grafiku i* $(x + 2)^2 + 3(x + 2)$
c *grafiku i* $x^2 + 3x - 3$ **b** *grafiku i* $(0.5x)^2 + 3(0.5x)$

Ushtrime 9A

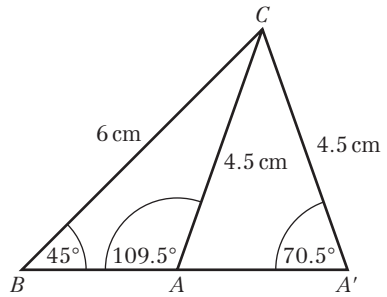
1 **a** *3.19 cm* **b** *1.73 cm ($\sqrt{3}$ cm)* **c** *9.85 cm*
d *4.31 cm* **e** *6.84 cm* **f** *9.80 cm*
2 **a** *108(.2)°* **b** *90°* **c** *60°*
d *52.6°* **e** *137°* **f** *72.2°*
3 *192 km*
4 *11.2 km*
5 *128.5° ose 031.5° (Këndi BAC = 48.5°)*
6 *302 jard (301.5…)*
7 *Me anë të rregullës së kosinusi* $\frac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4} = \frac{1}{8}$
8 *Me anë të rregullës së kosinusit* $\frac{2^2 + 3^2 - 4^2}{2 \times 2 \times 3} = -\frac{1}{4}$
9 *ACB = 22.3°*
10 *ABC = 108(.4)°*
11 *104(.48)°*
12 *x = 4.4 cm*
13 *x = 42 cm*
14 **a** $y^2 = (5 - x)^2 + (4 + x)^2 - 2(5 - x)(4 + x)\cos 120°$
$= 25 - 10x + x^2 + 16 + 8x + x^2 - 2(20 + x - x^2)\left(-\frac{1}{2}\right)$
$= x^2 - x + 61$
b *Minimumi* $AC^2 = 60.75$; *ai arrihet kur* $x = \frac{1}{2}$
15 **a** $\cos\angle ABC = \frac{x^2 + 5^2 - (10 - x)^2}{2x \times 5}$
$= \frac{20x - 75}{10x} = \frac{4x - 15}{2x}$
b *3.5*
16 *65.3°*
17 **a** *28.7 km* **b** *056.6°*

Ushtrime 9B

1 **a** *15.2 cm* **b** *9.57 cm* **c** *8.97 cm* **d** *4.61 cm*
2 **a** *x = 84°, y = 6.32*
b *x = 13.5, y = 16.6*
c *x = 85°, y = 13.9*
d *x = 80°, y = 6.22 (trekëndësh dybrinjënjëshëm)*
e *x = 6.27, y = 7.16*
f *x = 4.49, y = 7.49 (trekëndësh kënddrejtë)*
3 **a** *36.4°* **b** *35.8°* **c** *40.5°* **d** *130°*
4 **a** *48.1°* **b** *45.6°* **c** *14.8°* **d** *48.7°*
e *86.5°* **f** *77.4°*
5 **a** *1.41 cm ($\sqrt{2}$ cm)* **b** *1.93 cm*
6 *QPR = 50.6°, PQR = 54.4°*
7 **a** *x = 43.2°, y = 5.02 cm* **b** *x = 101°, y = 15.0 cm*
c *x = 6.58 cm, y = 32.1°* **d** *x = 54.6°, y = 10.3 cm*
e *x = 21.8°, y = 3.01* **f** *x = 45.9°, y = 3.87°*
8 **a** *6.52 km* **b** *3.80 km*
9 **a** *7.31 cm* **b** *1.97 cm*
10 **a** *66.3°* **b** *148 m*
11 *Nga rregulla e sinusit,* $x = \frac{4\sqrt{2}}{2 + \sqrt{2}}$; *duke racionalizuar*
$x = \frac{4\sqrt{2}(2 - \sqrt{2})}{2} = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$.
12 **a** *36.5 m*
b *Këndet maten nga niveli i tokës.*

Ushtrime 9C

1 **a** *70.5°, 109° (109.5°)*
b



2 **a** *x = 74.6°, y = 65.4°*
x = 105°, y = 34.6°
b *x = 59.8°, y = 48.4 cm*
x = 120°, y = 27.3cm
c *x = 56.8°, y = 4.37 cm*
x = 23.2°, y = 2.06 cm
3 **a** *5 cm (ACB = 90°)* **b** *24.6°*
c *45.6°, 134(.4)°*
4 *2.97 cm*
5 *Në një trekëndësh ABC = 101° (100.9°); në tjetrin BAC = 131° (130.9°)*
6 **a** *62.0°* **b** *Lëvizja është simetrike*

Ushtrime 9D

1 **a** *23.7 cm²* **b** *4.31 cm²* **c** *20.2 cm²*
2 **a** *x = 41.8° ose 138(.2)°*
b *x = 26.7° ose 153(.3)°*
c *x = 60° ose 120°*
3 *275(.3) m (brinja e tretë = 135.3 m)*
4 *3.58*
5 **a** *Syprina* $= \frac{1}{2}(x + 2)(5 - x)\sin 30°$
$= \frac{1}{2}(10 + 3x - x^2) \times \frac{1}{2}$
$= \frac{1}{4}(10 + 3x - x^2)$
b *Maksimumi* $A = 3\frac{1}{16}$ *kur* $x = 1\frac{1}{2}$
6 **a** $\frac{1}{2}x(5 + x)\sin 150° = \frac{15}{4}$
$\frac{1}{2}(5x + x^2) \times \frac{1}{2} = \frac{15}{4}$
$5x + x^2 = 15$
$x^2 + 5x - 15 = 0$
b *2.11*

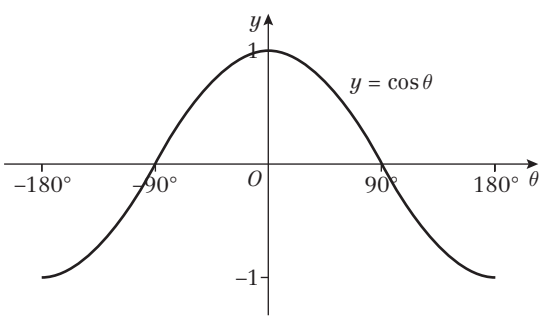
Ushtrime 9E

1 **a** *x = 37.7°, y = 86.3°, z = 6.86*
b *x = 48°, y = 19.5, z = 14.6*
c *x = 30°, y = 11.5, z = 11.5*
d *x = 21.0°, y = 29.0°, z = 8.09*
e *x = 93.8°, y = 56.3°, z = 29.9°*
f *x = 97.2°, y = 41.4°, z = 41.4°*
g *x = 45.3°, y = 94.7°, z = 14.7*
ose x = 135°, y = 5.27°, z = 1.36
h *x = 7.07, y = 73.7°, z = 61.3°*
or x = 7.07, y = 106°, z = 28.7°
i *x = 49.8°, y = 9.39, z = 37.0°*
2 **a** *ACB = 32.4°, ABC = 108°, AC = 15.1 cm*
Syprina= 41.3 cm²
b *BAC = 41.5°, ABC = 28.5°, AB = 9.65 cm*
Syprina= 15.7 cm²

3 **a** *8 km* **b** *060°*
4 *107 km*
5 *12 km*
6 **a** *5.44* **b** *7.95* **c** *36.8°*
7 **a** $AB + BC > AC \Rightarrow x + 6 > 7 \Rightarrow x > 1;$
$AC + AB > BC \Rightarrow 11 > x + 2 \Rightarrow x < 9$
b **i** $x = 6.08$ *nga* $x^2 = 37$
Syprina$= 14.0\ cm^2$
ii $x = 7.23$ *nga* $x^2 - 4(\sqrt{2} - 1)x - (29 + 8\sqrt{2}) = 0$
Syprina$= 13.1\ cm^2$
8 **a** $x = 4$ **b** $4.68\ cm^2$
9 $AC = 1.93\ cm$
10 **a** $AC^2 = (2 - x)^2 + (x + 1)^2 - 2(2 - x)(x + 1)\cos 120°$
$= (4 - 4x + x^2) + (x^2 + 2x + 1) - 2(-x^2 + x + 2)\left(-\frac{1}{2}\right)$
$= x^2 - x + 7$
b $\frac{1}{2}$
11 $4\sqrt{10}$
12 $AC = 1\frac{2}{3}cm$ *dhe* $BC = 6\frac{1}{3}cm$
Syprina$= 5.05\ cm^2$
13 **a** *61.3°* **b** $78.9\ cm^2$
14 **a** $DAB = 136.3°$, $BCD = 50.1°$
b $13.1\ m^2$
c *5.15 m*
15 $34.2\ cm^2$

Ushtrime 9F

1



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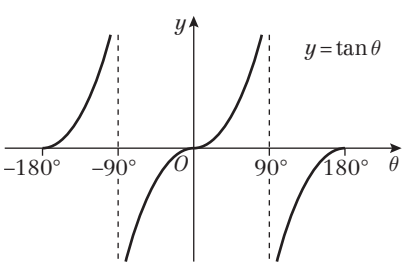

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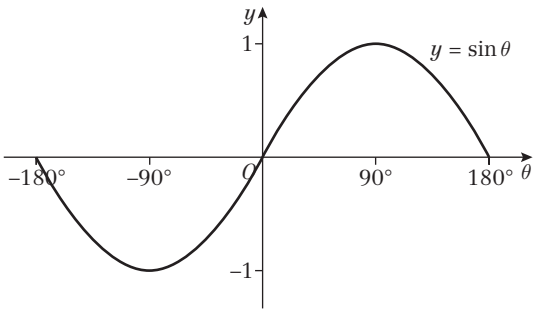
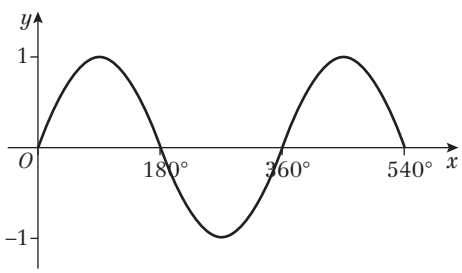


4 **a** $-30°$
b **i** $-120°$ **ii** $-60°, 120°$
c **i** $135°$ **ii** $-45°, -135°$

KAPITULLI 10
Kontrolli i njohurive të mëparshme

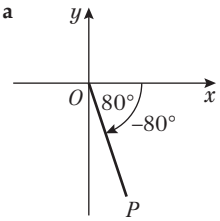
1 **a**



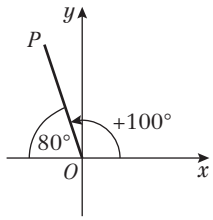
b *4*
c *143.1°, 396.9°, 503.1°*
2 **a** *57.7°* **b** *73.0°*
3 **a** $x = 11$ **b** $x = \frac{9}{4}$ **c** $x = -44.4°$
4 **a** $x = 1$ *ose* $x = 3$
b $x = 1$ *ose* $x = -9$
c $x = \dfrac{3 \pm \sqrt{65}}{4}$

Ushtrime 10A

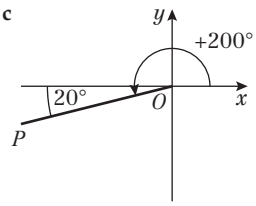
1 **a**



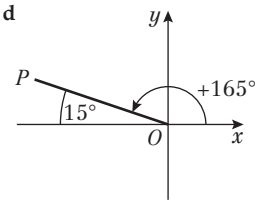
b



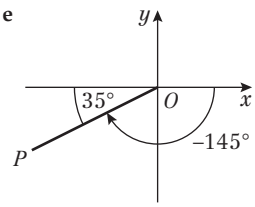
c



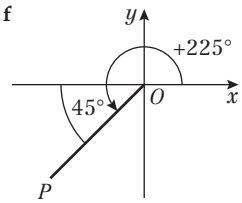
d



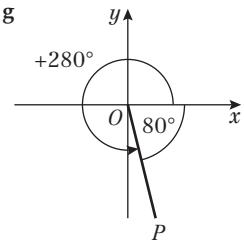
e



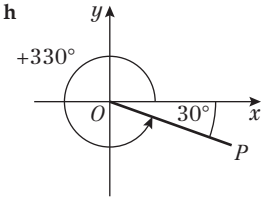
f



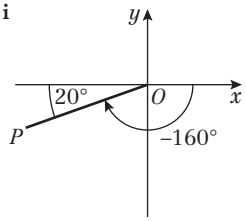
g



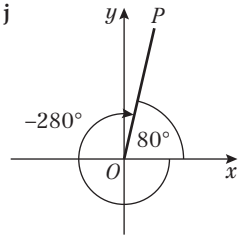
h



i



j



2 **a** *I pari* **b** *I dyti* **c** *I dyti*
d *I treti* **e** *I treti*

3 **a** -1 **b** 1 **c** 0 **d** -1 **e** -1
f 0 **g** 0 **h** 0 **i** 0 **j** 0

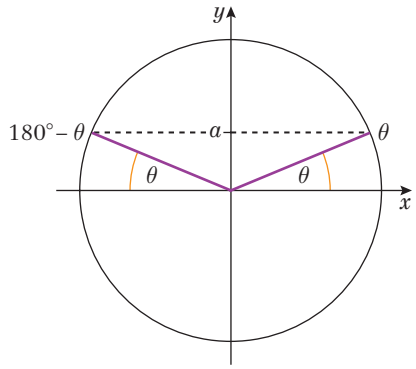
4 **a** $-\sin 60°$ **b** $-\sin 80°$ **c** $\sin 20°$
d $-\sin 60°$ **e** $\sin 80$ **f** $-\cos 70°$
g $-\cos 80°$ **h** $\cos 50°$ **i** $-\cos 20°$
j $-\cos 5°$ **k** $-\tan 80°$ **l** $-\tan 35°$
m $-\tan 30°$ **n** $\tan 5°$ **o** $\tan 60°$

5 **a** $-\sin\theta$ **b** $-\sin\theta$ **c** $-\sin\theta$
d $\sin\theta$ **e** $-\sin\theta$ **f** $\sin\theta$
g $-\sin\theta$ **h** $-\sin\theta$ **i** $\sin\theta$

6 **a** $-\cos\theta$ **b** $-\cos\theta$ **c** $\cos\theta$
d $-\cos\theta$ **e** $\cos\theta$ **f** $-\cos\theta$
g $-\tan\theta$ **h** $-\tan\theta$ **i** $\tan\theta$
j $\tan\theta$ **k** $-\tan\theta$ **l** $\tan\theta$

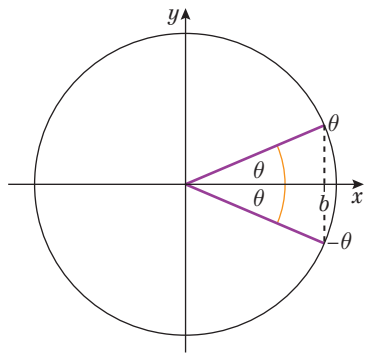
Sfidë

a



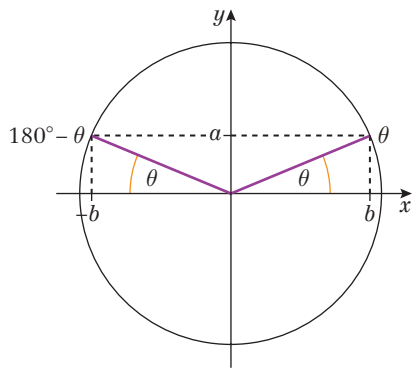
$\sin\theta = \sin(180° - \theta) = a$

b



$\cos\theta = \cos(-\theta) = b$

c



$\tan\theta = \frac{a}{b}$; $\tan(180° - \theta) = \frac{a}{-b} = -\tan\theta$

Ushtrime 10B

1 **a** $\frac{\sqrt{2}}{2}$ **b** $-\frac{\sqrt{3}}{2}$ **c** $-\frac{1}{2}$ **d** $\frac{\sqrt{3}}{2}$
e $\frac{\sqrt{3}}{2}$ **f** $-\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $-\frac{\sqrt{2}}{2}$
i $-\frac{\sqrt{3}}{2}$ **j** $-\frac{\sqrt{2}}{2}$ **k** -1 **l** -1
m $-\frac{\sqrt{3}}{3}$ **n** $-\sqrt{3}$ **o** $\sqrt{3}$

Sfidë

a **i** $\sqrt{3}$ **ii** 2 **iii** $\sqrt{2+\sqrt{3}}$ **iv** $\sqrt{2+\sqrt{3}} - \sqrt{2}$

b $15°$

c **i** $\frac{\sqrt{2+\sqrt{3}} - \sqrt{2}}{2}$ **ii** $\frac{\sqrt{2+\sqrt{3}}}{2}$

Ushtrime 10C

1 **a** $\sin^2\frac{\theta}{2}$ **b** 5 **c** $-\cos^2 A$
d $\cos\theta$ **e** $\tan x$ **f** $\tan 3A$
g 4 **h** $\sin^2\theta$ **i** 1

2 $1\frac{1}{2}$

3 $3\tan y$

4 **a** $1-\sin^2\theta$ **b** $\frac{\sin^2\theta}{1-\sin^2\theta}$ **c** $\sin\theta$
d $\frac{1-\sin^2\theta}{\sin\theta}$ **e** $1-2\sin^2\theta$

5 (Një shembull skicë e vërtetimit është dhënë)
a Ana e majtë $=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$
$=1+2\sin\theta\cos\theta$
= Ana e djathtë
b Ana e majtë $=\frac{1-\cos^2\theta}{\cos\theta}=\frac{\sin^2\theta}{\cos\theta}=\sin\theta\times\frac{\sin\theta}{\cos\theta}$
$=\sin\theta\tan\theta$ = Ana e djathtë
c Ana e majtë $=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}$
$=\frac{1}{\sin x\cos x}$ = Ana e djathtë
d Ana e majtë $=\cos^2 A-(1-\cos^2 A)=2\cos^2 A-1$
$=2(1-\sin^2 A)-1=1-2\sin^2 A$ = Ana e djathtë
e Ana e majtë $=(4\sin^2\theta-4\sin\theta\cos\theta+\cos^2\theta)$
$+(\sin^2\theta+4\sin\theta\cos\theta+\cos^2\theta)$
$=5(\sin^2\theta+\cos^2\theta)=5$ = Ana e djathtë
f Ana e majtë $=2-(\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta)$
$=2(\sin^2\theta+\cos^2\theta)-(\sin^2\theta-2\sin\theta\cos\theta$
$+\cos^2\theta)$
$=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$
$=(\sin\theta+\cos\theta)^2$ = Ana e djathtë
g Ana e majtë $=\sin^2 x(1-\sin^2 y)-(1-\sin^2 x)\sin^2 y$
$=\sin^2 x-\sin^2 y$ = Ana e djathtë

6 **a** $\sin\theta=\frac{5}{13}$, $\cos\theta=\frac{12}{13}$
b $\sin\theta=\frac{4}{5}$, $\tan\theta=-\frac{4}{3}$
c $\cos\theta=\frac{24}{25}$, $\tan\theta=-\frac{7}{24}$

7 **a** $-\frac{\sqrt{5}}{3}$ **b** $-\frac{2\sqrt{5}}{5}$

8 **a** $-\frac{\sqrt{3}}{2}$ **b** $\frac{1}{2}$

9 **a** $-\frac{\sqrt{7}}{4}$ **b** $-\frac{\sqrt{7}}{3}$

10 **a** $x^2+y^2=1$
b $4x^2+y^2=4$ (ose $x^2+\frac{y^2}{4}=1$)
c $x^2+y=1$
d $x^2=y^2(1-x^2)$ (ose $x^2+\frac{x^2}{y^2}=1$)
e $x^2+y^2=2$ (ose $\frac{(x+y)^2}{4}+\frac{(x-y)^2}{4}=1$)

11 **a** Nga teorema e kosinusit $\cos B=\frac{8^2+12^2-10^2}{2\times 8\times 12}=\frac{9}{16}$
b $\frac{\sqrt{175}}{16}$

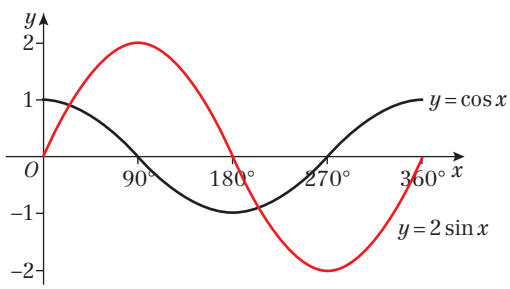
12 **a** Nga teorema e sinusit: $\sin Q=\frac{\sin 30}{6}\times 8=\frac{2}{3}$
b $=\frac{\sqrt{5}}{3}$

Ushtrime 10D

1 **a** −63.4° **b** 116.6°, 296.6°
2 **a** 66.4° **b** 66.4°, 113.6°, 246.4°, 293.6°
3 **a** 270° **b** 60°, 240°
c 60°, 300° **d** 15°, 165°
e 140°, 220° **f** 135°, 315°
g 90°, 270° **h** 230°, 310°
4 **a** 45.6°, 134.4° **b** 135°, 225°
c 132°, 228° **d** 229°, 311°
e 8.13°, 188° **f** 61.9°, 242°
g 105°, 285° **h** 41.8°, 318°
5 **a** 30°, 210° **b** 135°, 315°
c 53.1°, 233° **d** 56.3°, 236°
e 54.7°, 235° **f** 148°, 328°
6 **a** −120°, −60°, 240°, 300° **b** −171°, −8.63°
c −144°, 144° **d** −327°, −32.9°
e 150°, 330°, 510°, 690° **f** 251°, 431°

7 **a** $\tan x$ duhet të jetë $\frac{2}{3}$
b Ngritja në katror e të dyja anëve sjell zgjidhje të tjera.
c −146.3°, 33.7°

8 **a**



b 2 **c** 26.6°, 206.6°

9 71.6°, 108.4°, 251.6°, 288.4°

10 **a** $4\sin^2 x-3(1-\sin^2 x)=2$.
Rishkruaj dhe gjej $7\sin^2 x=5$
b 57.7°, 122.3°, 237.7°, 302.3°

11 **a** $2\sin^2 x+5(1-\sin^2 x)=1$.
Rishkruaj dhe gjej $3\sin^2 x=4$
b $\sin x>1$

Ushtrime 10F

1 **a** 60°, 120°, 240°, 300°
b 45°, 135°, 225°, 315°
c 0°, 180°, 199°, 341°, 360°
d 77.0°, 113°, 257°, 293°
e 60°, 300°
f 204°, 336°
g 30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°

2 **a** ±45°, ±135° **b** −180°, −117°, 0°, 63.4°, 180°
c ±114° **d** 0°, ±75.5°, ±180°

3 **a** 72°, 144° **b** 0°, 60°
c Nuk ka zgjidhje në bashkësi.

4 **a** ±41.8°, ±138° **b** 38.2°, 142°

5 60°, 75.5°, 284.5°, 300°

6 48.2°, 131.8°, 228.2°, 311.8°

7 $2\cos^2 x+\cos x-6=(2\cos x-3)(\cos x+2)$
Nuk ka zgjidhje për $\cos x=-2$ ose $\cos x=\frac{3}{2}$

8 a $1 - \sin^2 x = 2 - \sin x$
Rishkruaj dhe gjen $\sin^2 x - \sin x + 1 = 0$
b Ekuacioni nuk ka zgjidhje reale kur $b^2 - 4ac < 0$
9 a $p = 1, q = 5$
b 72.8°, 129.0°, 252.8°, 309.0°, 432.8°, 489.0°

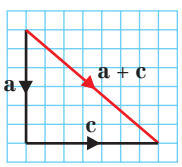
Sfidë
1 −180°, −60°, 60°, 180°
2 0°, 90°, 180°, 270°, 360°

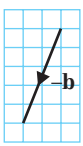
KAPITULLI 11

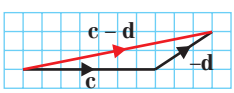
Kontrolli i njohurive të mëparshme

1 a $\binom{4}{2}$ b $\binom{5}{-2}$ c $\binom{-1}{-3}$
2 a $\frac{7}{9}$ b $\frac{2}{9}$ c $\frac{7}{2}$
3 a 123.2° b 13.6 c 5.3 d 21.4°

Ushtrime 11A

1 a


b


c


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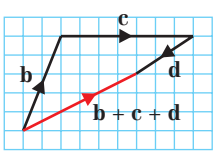
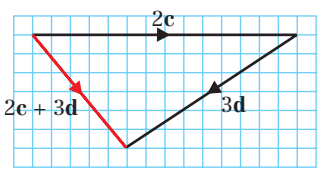
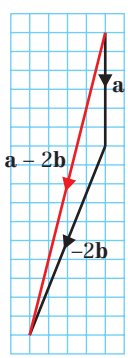
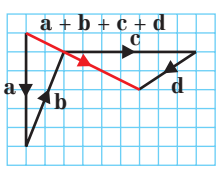


e


f


g


2 a 2b b d c b
d 2b e d + b f d + b
g −2d h −b i 2d + b
j −b + 2d k −b + d l −d − b
3 a 2m b 2p c m
d m e p + m f p + m
g p + 2m h p − m i −m −p
j −2m + p k −2p + m l −m − 2p
4 a d − a b a + b + c
c a + b − d d a + b + c − d

5 a 2a + 2b b a + b c b − a
6 a b b b − 3a c a − b
d 2a − b

7 a $\overrightarrow{OB} = \mathbf{a} + \mathbf{b}$ b $\overrightarrow{OP} = \frac{5}{8}(\mathbf{a} + \mathbf{b})$ c $\overrightarrow{AP} = \frac{5}{8}\mathbf{b} - \frac{3}{8}\mathbf{a}$
8 a Po ($\lambda = 2$) b Po ($\lambda = 4$) c Jo
d Po ($\lambda = -1$) e Po ($\lambda = -3$) f JO
9 a i b − a ii $\frac{1}{2}$a iii $\frac{1}{2}$b iv $\frac{1}{2}$b − $\frac{1}{2}$a
b $\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{PQ} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$ pra PQ është paralele me BC.
10 a i 2b ii a − b
b $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{OC} = 3\mathbf{b}$ pra AB është paralele me OC.
11 1.2

Ushtrime 11B

1 $\mathbf{v}_1$: 8i, $\binom{8}{0}$ $\mathbf{v}_2$: 9i + 3j, $\binom{9}{3}$ $\mathbf{v}_3$: −4i + 2j, $\binom{-4}{2}$
$\mathbf{v}_4$: 3i + 5j, $\binom{3}{5}$ $\mathbf{v}_5$: −3i − 2j, $\binom{-3}{-2}$ $\mathbf{v}_6$: −5j, $\binom{0}{-5}$
2 a 8i + 12j b i + 1.5j c −4i + j
d 10i + j e −2i + 11j f −2i − 10j
g 14i − 7j h −8i + 9j
3 a $\binom{45}{35}$ b $\binom{4}{0.5}$ c $\binom{12}{3}$
d $\binom{-1}{16}$ e $\binom{-21}{-29}$ f $\binom{10}{2}$
4 a $\lambda = 5$ b $\mu = -\frac{3}{2}$
5 a $\lambda = \frac{1}{3}$ b $\mu = -1$
c $s = -1$ d $t = -\frac{1}{17}$
6 i − j
7 a $\overrightarrow{AC} = 5\mathbf{i} - 4\mathbf{j} = \binom{5}{-4}$ b $\overrightarrow{AP} = 3\mathbf{i} - \frac{12}{5}\mathbf{j} = \binom{3}{-\frac{12}{5}}$
c $\overrightarrow{OP} = 5\mathbf{i} + \frac{8}{5}\mathbf{j} = \binom{5}{\frac{8}{5}}$
8 $j = 4, k = 11$
9 $p = 3, q = 2$
10 a $p = 5$ b 8i − 12j

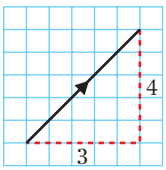
Ushtrime 11C

1 a 5 b 10 c 13
d 4.47 (3 sh.dh.) e 5.83 (3 sh.dh.) f 8.06 (3 sh.dh.)
g 5.83 (3 sh.dh.) h 4.12 (3 sh.dh.)
2 a $\sqrt{26}$ b $5\sqrt{2}$ c $\sqrt{101}$
3 a $\frac{1}{5}\binom{4}{3}$ b $\frac{1}{13}\binom{5}{-12}$
c $\frac{1}{25}\binom{-7}{24}$ d $\frac{1}{\sqrt{10}}\binom{1}{-3}$
4 a 53.1° mbi b 53.1° nën
c 67.4° mbi d 63.4° mbi
5 a 149° djathtas b 29.7° djathtas
c 31.0° majtas d 104° majtas

6 a $\frac{15\sqrt{2}}{2}$ i + $\frac{15\sqrt{2}}{2}$j b 7.52i + 2.74 j, $\binom{7.52}{2.74}$

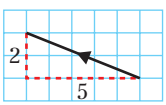
c $18.1\mathbf{i} - 8.45\mathbf{j}$, $\begin{pmatrix}18.1\\-8.45\end{pmatrix}$ **d** $\frac{5\sqrt{3}}{2}\mathbf{i} - 2.5\mathbf{j}$, $\begin{pmatrix}\frac{5\sqrt{3}}{2}\\-2.5\end{pmatrix}$

7 **a** $|3\mathbf{i} + 4\mathbf{j}| = 5$, 53.1° mbi



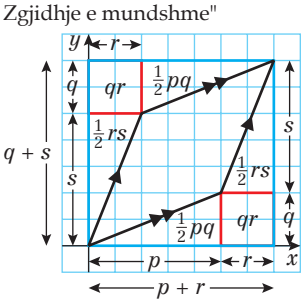
b $|2\mathbf{i} - \mathbf{j}| = \sqrt{5}$, 26.6° nën

c $|-5\mathbf{i} + 2\mathbf{j}| = \sqrt{29}$, 158.2° mbi



8 $k = \pm 6$
9 $p = \pm 8$, $q = 6$
10 **a** 36.9° **b** 33.7° **c** 70.6°
11 **a** 67.2° **b** 19.0

Sfidë
Zgjidhje e mundshme"



Syprina e paralelogramit = syprina e drejtkëndëshit të madh − 2(syprina e drejtkëndëshit të vogël) − 2 (syprina e trekëndëshit 1) − 2(syprina e trekëndëshit 2)

Syprina e paralelogramit = $(p + r)(q + s) - 2qr - 2(\frac{1}{2}pq)$
$- 2(\frac{1}{2}rs) = ps - qr$

Ushtrime 11D

1 **a** **i** $\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} + 5\mathbf{j}$, $\overrightarrow{OC} = -2\mathbf{i} + 6\mathbf{j}$
ii $\mathbf{i} + 6\mathbf{j}$ **iii** $-5\mathbf{i} + 7\mathbf{j}$
b **i** $\sqrt{40} = 2\sqrt{10}$ **ii** $\sqrt{37}$ **iii** $\sqrt{74}$

2 **a** $-\mathbf{i} + 5\mathbf{j}$ ose $\begin{pmatrix}-1\\5\end{pmatrix}$
b **i** 5 **ii** $\sqrt{13}$ **iii** $\sqrt{26}$

3 **a** $-\mathbf{i} - 9\mathbf{j}$ ose $\begin{pmatrix}-1\\-9\end{pmatrix}$
b **i** $\sqrt{82}$ **ii** 5 **iii** $\sqrt{61}$

4 **a** $-2\mathbf{a} + 2\mathbf{b}$ **b** $-3\mathbf{a} + 2\mathbf{b}$ **c** $-2\mathbf{a} + \mathbf{b}$

5 $\begin{pmatrix}7\\9\end{pmatrix}$ ose $\begin{pmatrix}9\\3\end{pmatrix}$

6 **a** $2\mathbf{i} + 8\mathbf{j}$ **b** $2\sqrt{17}$

7 $\frac{3\sqrt{5}}{5}$

Sfidë
$\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{j}$ ose $\overrightarrow{OB} = \frac{46}{13}\mathbf{i} + \frac{9}{13}\mathbf{j}$

Ushtrime 11E

1 $\overrightarrow{XY} = \mathbf{b} - \mathbf{a}$ dhe $\overrightarrow{YZ} = \mathbf{c} - \mathbf{b}$, pra $\mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{b}$.
Rrjedhimisht $\mathbf{a} + \mathbf{c} = 2\mathbf{b}$.

2 **a** **i** $2\mathbf{r}$ **ii** $\mathbf{r}$
b Brinjët e trekëndëshit OAB janë sa dyfishi i gjatësisë së brinjëve të trekëndëshit PAQ dhe këndi A është i përbashkët te të dy SAS.

3 **a** $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$
b $\overrightarrow{AN} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$ $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{NB} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$
$AN : NB = 1 : 2$.

4 **a** $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}$
b $\overrightarrow{AP} = -\mathbf{a} + \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c} = \frac{2}{5}(\mathbf{c} - \mathbf{a})$,
$\overrightarrow{PC} = \mathbf{c} - (\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}) = \frac{3}{5}(\mathbf{c} - \mathbf{a})$ pra $AP : PC = 2 : 3$

5 **a** $\sqrt{26}$ **b** $2\sqrt{2}$ **c** $3\sqrt{2}$
d $\angle BAC = 56°$, $\angle ABC = 34°$, $\angle ACB = 90°$

6 **a** $\overrightarrow{OR} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$,
$\overrightarrow{OS} = 3\ \overrightarrow{OR} = 3(\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}) = 2\mathbf{a} + \mathbf{b}$
b $\overrightarrow{TP} = \overrightarrow{TO} + \overrightarrow{OP} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{PS} = \overrightarrow{OP} + \overrightarrow{OS} = -\mathbf{a} + 2\mathbf{a} + \mathbf{b} = \mathbf{a} + \mathbf{b}$
$\overrightarrow{TP}$ është paralel (dhe i barabartë) me $\overrightarrow{PS}$ dhe ata kanë një pikë të përbashkët, P, pra T, P dhe S ndodhen në një vijë të drejtë.

Sfidë:

a $\overrightarrow{PR} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{PX} = j(\mathbf{b} - \mathbf{a}) = -j\mathbf{a} + j\mathbf{b}$

b $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}\mathbf{b}$, $\overrightarrow{PX} = -\mathbf{a} + k(\mathbf{a} + \frac{1}{2}\mathbf{b}) = (k - 1)\mathbf{a} + \frac{1}{2}k\mathbf{b}$

c Koeficientet e $\mathbf{a}$ dhe $\mathbf{b}$ duhet të jenë të njëjtë në të dy shprehjet $\overrightarrow{PX}$
Koeficienti i $\mathbf{a}$: $k - 1 = -j$; Koeficienti i $\mathbf{b}$: $j = \frac{1}{2}k$

d Duke zgjidhur njëherësh kemi $j = \frac{1}{3}$ dhe $k = \frac{2}{3}$

e $\overrightarrow{PX} = \frac{1}{3}\overrightarrow{PR}$
Nga simetria, $\overrightarrow{PX} = \overrightarrow{YR} = \overrightarrow{XY}$, pra ON dhe OM ndajnë PR në 3 pjesë të barabarta.

KAPITULLI 12

Kontrolli i njohurive të mëparshme

1 **a** 5 **b** $-\frac{2}{3}$ **c** $\frac{1}{3}$

2 **a** x^{10} **b** $x^{\frac{2}{3}}$ **c** x^{-1} **d** $x^{\frac{3}{4}}$

3 **a** $y = \frac{1}{2}x - 2$ **b** $y = -\frac{1}{2}x + 8\frac{1}{2}$ **c** $y = -\frac{1}{4}x + 7\frac{1}{2}$

4 $y = -\frac{1}{2}x$

Ushtrime 12A

1 **a**

koordinata x	−1	0	1	2	3
Vlerësimi për koeficientin këndor të vijës	−4	−2	0	2	4

b Gradienti = $2p - 2$ **c** 1

2 **a** $\sqrt{1 - 0.6^2} = \sqrt{0.64} = 0.8$

b Gradienti = −0.75

c **i** −1.21 (3 sh.dh.) **ii** −1 **iii** −0.859 (3 sh.dh.)

d Kur pikat e tjera janë afër me A, atëherë koeficienti këndor tenton te −0.75.

3 **a** **i** 7 **ii** 6.5 **iii** 6.1 **iv** 6.01 **v** $h + 6$

b Koeficientin këndor i tangjentes = 6

4 **a** **i** 9 **ii** 8.5 **iii** 8.1 **iv** 8.01 **v** $8 + h$

b Koeficientin këndor i tangjentes = 8

Ushtrime 12B

1 **a** $f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^2 - 2^2}{h}$

$= \lim_{h\to 0} \frac{4h + h^2}{h} = \lim_{h\to 0} (4 + h) = 4$

b $f'(-3) = \lim_{h\to 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h\to 0} \frac{(-3+h)^2 - 3^2}{h}$

$= \lim_{h\to 0} \frac{-6h + h^2}{h} = \lim_{h\to 0}(-6 + h) = -6$

c $f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h^2 - 0^2}{h} = \lim_{h\to 0} h = 0$

d $f'(50) = \lim_{h\to 0} \frac{f(50+h) - f(50)}{h} = \lim_{h\to 0} \frac{(50+h)^2 - 50^2}{h}$

$= \lim_{h\to 0} \frac{100h + h^2}{h} = \lim_{h\to 0}(100 + h) = 100$

2 **a** $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} (2x + h)$

b Nëse $h \to 0$, $f'(x) = \lim_{h\to 0} (2x + h) = 2x$

3 **a** $g = \lim_{h\to 0} \frac{(-2+h)^3 - (-2)^3}{h}$

$= \lim_{h\to 0} \frac{-8 + 3(-2)^2h + 3(-2)h^2 + h^3 + 8}{h}$

$= \lim_{h\to 0} \frac{12h - 6h^2 + h^3}{h} = \lim_{h\to 0} (12 - 6h + h^2)$

b $g = 12$

4 **a** Koeficientin këndor i $AB = \frac{(-1+h)^3 - 5(-1+h) - 4}{(-1+h) - (-1)}$

$= \frac{-1 + 3h - 3h^2 + h^3 + 5 - 5h - 4}{h}$

$= \frac{h^3 - 3h^2 - 2h}{h} = h^2 - 3h - 2$

b Koeficientin këndor = −2

5 $\frac{dy}{dx} = \lim_{h\to 0} \frac{6(x+h) - 6x}{h} = \lim_{h\to 0} \frac{6h}{h} = 6$

6 $\frac{dy}{dx} = \lim_{h\to 0} \frac{4(x+h)^2 - 4x^2}{h} = \lim_{h\to 0} \frac{8xh + 4h^2}{h}$

$= \lim_{h\to 0} (8x + 4h) = 8x$

7 $\frac{dy}{dx} = \lim_{h\to 0} \frac{a(x+h)^2 - ax^2}{h} = \lim_{h\to 0} \frac{(a-a)x^2 + 2axh + ah^2}{h}$

$= \lim_{h\to 0} \frac{2axh + ah^2}{h} = \lim_{h\to 0} (2ax + ah) = 2ax$

Sfidë

a $f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{x - (x+h)}{xh(x+h)} = \lim_{h\to 0} \frac{-1}{x(x+h)}$

$= \lim_{h\to 0} \frac{-1}{x^2 + xh}$

b $f'(x) = \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2 + xh} = \frac{-1}{x^2 + 0} = -\frac{1}{x^2}$

Ushtrime 12C

1 **a** $7x^6$ **b** $8x^7$ **c** $4x^3$ **d** $\frac{1}{3}x^{-\frac{2}{3}}$
e $\frac{1}{4}x^{-\frac{3}{4}}$ **f** $\frac{1}{3}x^{-\frac{2}{3}}$ **g** $-3x^{-4}$ **h** $-4x^{-5}$
i $-2x^{-3}$ **j** $-5x^{-6}$ **k** $-\frac{1}{2}x^{-\frac{3}{2}}$ **l** $-\frac{1}{3}x^{-\frac{4}{3}}$
m $9x^8$ **n** $5x^4$ **o** $3x^2$ **p** $-2x^{-3}$
q 1 **r** $3x^2$

2 **a** $6x$ **b** $54x^8$ **c** $2x^3$ **d** $5x^{-\frac{3}{4}}$
e $\frac{15}{2}x^{\frac{1}{4}}$ **f** $-10x^{-2}$ **g** $6x^2$ **h** $-\frac{1}{2x^5}$
i $x^{-\frac{3}{2}}$ **j** $\frac{15}{2}\sqrt{x}$

3 **a** $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** 3 **d** 2

4 $\frac{dy}{dx} = \frac{3\sqrt{x}}{2}$

Ushtrime 12D

1 **a** $4x - 6$ **b** $x + 12$ **c** $8x$ **d** $16x + 7$
e $4 - 10x$

2 **a** 12 **b** 6 **c** 7 **d** $2\frac{1}{2}$
e −2 **f** 4

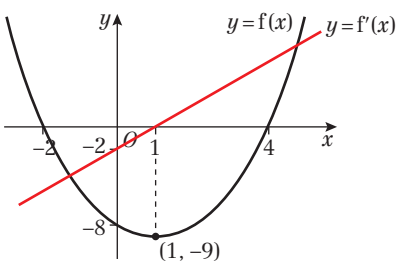
3 4, 0

4 (−1, −8)

5 1, −1

6 6, −4

7 **a, b**



c At the turning point, the gradient of $y = f(x)$ is zero, i.e. $f'(x) = 0$.

Ushtrime 12E

1 **a** $4x^3 - x^{-2}$ **b** $10x^4 - 6x^{-3}$ **c** $9x^{\frac{1}{2}} - x^{-\frac{3}{2}}$

2 **a** 0 **b** $11\frac{1}{2}$

3 **a** $(2\frac{1}{2}, -6\frac{1}{4})$ **b** (4, –4) dhe (2, 0)
c (16, –31) **d** $(\frac{1}{2}, 4)$ $(-\frac{1}{2}, -4)$

4 **a** $x^{-\frac{1}{2}}$ **b** $-6x^{-3}$ **c** $-x^{-4}$
d $\frac{4}{3}x^3 - 2x^2$ **e** $\frac{1}{2}x^{-\frac{1}{2}} - 6x^{-4}$ **f** $\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$
g $-3x^{-2}$ **h** $3 + 6x^{-2}$ **i** $5x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$
j $3x^2 - 2x + 2$ **k** $12x^3 + 18x^2$ **l** $24x - 8 + 2x^{-2}$

5 **a** 1 **b** $\frac{2}{9}$ **c** –4 **d** 4

6 $-\frac{3}{4}\sqrt{2}$

7 **a** $512 - 2304x + 4608x^2$

b $f'(x) \approx \frac{d}{dx}(512 - 2304x + 4608x^2)$
$= -2304 + 2 \times 4608x$
$= 9216x - 2304$

Ushtrime 12F

1 **a** $y + 3x - 6 = 0$ **b** $4y - 3x - 4 = 0$
c $3y - 2x - 18 = 0$ **d** $y = x$
e $y = 12x + 14$ **f** $y = 16x - 22$

2 **a** $7y + x - 48 = 0$ **b** $17y + 2x - 212 = 0$

3 $(1\frac{2}{9}, 1\frac{8}{9})$

4 $y = -x$, $4y + x - 9 = 0$; (–3, 3)

5 $y = -8x + 10$, $8y - x - 145 = 0$

6 $(-\frac{3}{4}, \frac{9}{8})$

Sfidë
L Ka ekuacion $y = 12x - 8$.

Ushtrime 12G

1 **a** $x \geq -\frac{4}{3}$ **b** $x \leq \frac{2}{3}$ **c** $x \leq -2$
d $x \leq 2, x \geq 3$ **e** $x \in \mathbb{R}$ **f** $x \in \mathbb{R}$
g $x \geq 0$ **h** $x \geq 6$

2 **a** $x \leq 4.5$ **b** $x \geq 2.5$
c $x \geq -1$ **d** $-1 \leq x \leq 2$
e $-3 \leq x \leq 3$ **f** $-5 \leq x < 0, 0 < x \leq 5$
g $0 < x \leq 9$ **h** $-2 \leq x \leq 0$

3 $f'(x) = -6x^2 - 3$
$x^2 \geq 0$ për të gjithë $x \in \mathbb{R}$, pra $-6x^2 - 3 \leq 0$ për të gjithë $x \in \mathbb{R}$.
$\therefore$ f (x) Është zvogëlues për të gjithë $x \in \mathbb{R}$.

4 **a** Çdo $p \geq 2$
b Jo. Mund të jetë çdo $p \geq 2$.

Ushtrime 12H

1 **a** $24x + 3$, 24
b $15 - 3x^{-2}$, $6x^{-3}$
c $\frac{9}{2}x^{-\frac{1}{2}} + 6x^{-3}$, $-\frac{9}{4}x^{-\frac{3}{2}} - 18x^{-4}$
d $30x + 2$, 30
e $-3x^{-2} - 16x^{-3}$, $6x^{-3} + 48x^{-4}$

2 Përshpejtimi $= \frac{3}{4}t^{-\frac{1}{2}} + \frac{3}{2}t^{-\frac{5}{2}}$

3 $\frac{3}{2}$

4 $-\frac{1}{2}$

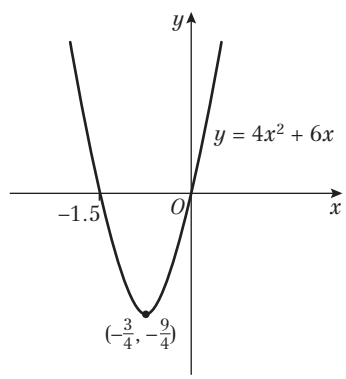
Ushtrime 12I

1 **a** –28 **b** –17 **c** $-\frac{1}{5}$

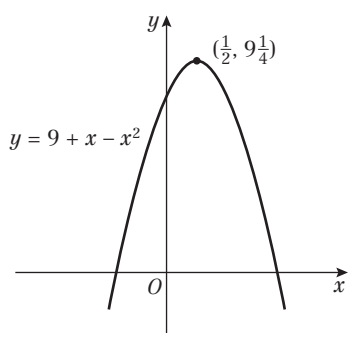
2 **a** 10 **b** 4 **c** 12.25

3 **a** $(-\frac{3}{4}, -\frac{9}{4})$ minimum
b $(\frac{1}{2}, 9\frac{1}{4})$ maksimum
c $(-\frac{1}{3}, 1\frac{5}{27})$ maksimum, (1, 0) minimum
d (3, –18) minimum, $(-\frac{1}{3}, \frac{14}{27})$ maksimum
e (1, 2) minimum, (–1, –2) maksimum
f (3, 27) minimum
g $(\frac{9}{4}, -\frac{9}{4})$ minimum
h $(2, -4\sqrt{2})$ minimum
i $(\sqrt{6}, -36)$ minimum, $(-\sqrt{6}, -36)$ minimum, (0, 0) maksimum

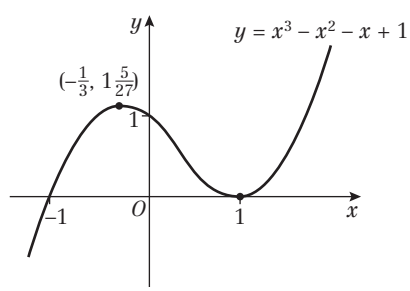
4 **a**



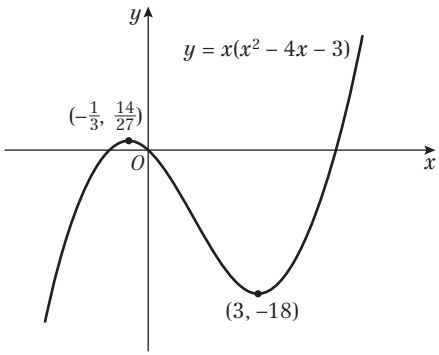
b



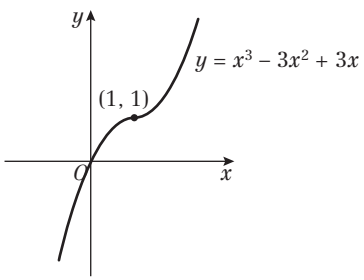
c



d



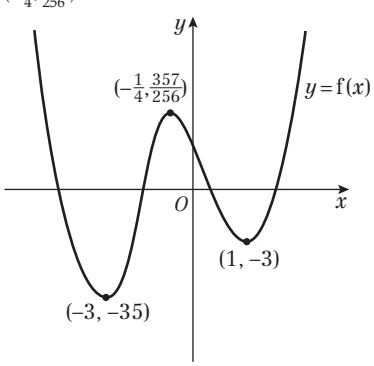
5 (1, 1) infleksion (Koeficientin këndor është pozitiv në një nga anët e pikës)



6 Vlera maksimale është 27; $f(x) \leqslant 27$

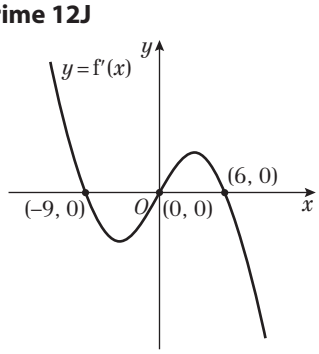
7 **a** (1, −3): minimum, (−3, −35): minimum, $(-\frac{1}{4}, \frac{357}{256})$: maksimum

b

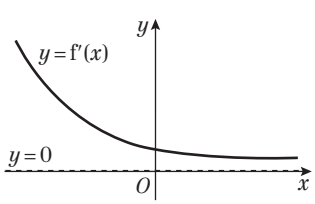


Ushtrime 12J

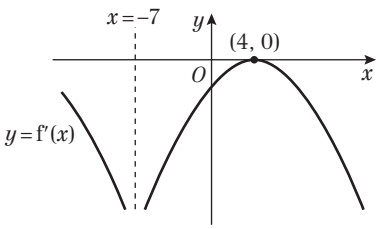
1 **a**



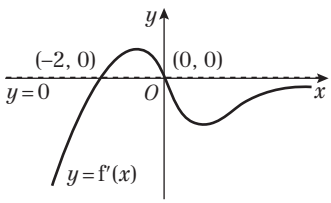
b



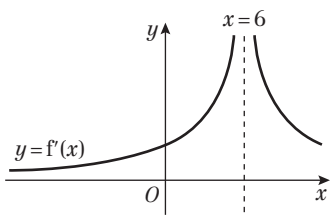
c



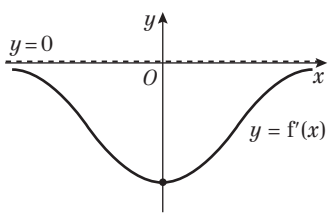
d



e

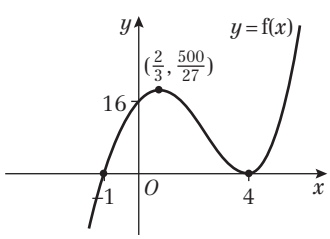


f



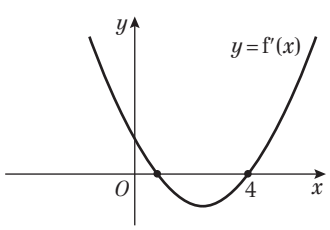
2 **a** $f(x) = x^3 - 7x^2 + 8x + 16$
$f'(x) = 3x^2 - 14x + 8 = (3x - 2)(x - 4)$

b



c (4, 0), $(\frac{2}{3}, 0)$ dhe (0, 8)

d



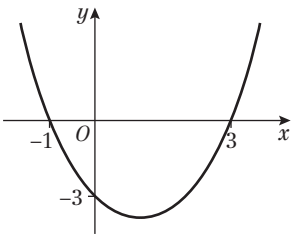
KAPITULLI 13

Kontrolli i njohurive të mëparshme

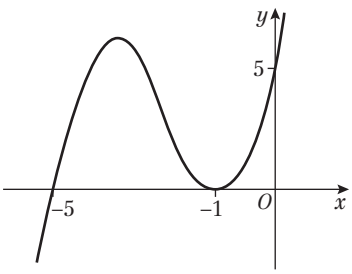
1 **a** $x^{\frac{5}{2}}$ **b** $2x^{\frac{3}{2}}$ **c** $x^{\frac{5}{2}} - \sqrt{x}$ **d** $x^{\frac{3}{2}} + 4x$

2 **a** $6x^2 + 3$ **b** $x - 1$ **c** $3x^2 + 2x$ **d** $-\frac{1}{x^2} - 3x^2$

3 **a**



b



Ushtrime 13A

1 **a** $y = \frac{1}{6}x^6 + c$ **b** $y = 2x^5 + c$
c $y = x^{-1} + c$ **d** $y = 2x^{-2} + c$
e $y = \frac{3}{5}x^{\frac{5}{3}} + c$ **f** $y = \frac{8}{3}x^{\frac{3}{2}} + c$
g $y = -\frac{2}{7}x^7 + c$ **h** $y = 2x^{\frac{1}{2}} + c$
i $y = -10x^{-\frac{1}{2}} + c$ **j** $y = \frac{9}{2}x^{\frac{4}{3}} + c$
k $y = 3x^{12} + c$ **l** $y = 2x^{-7} + c$
m $y = -9x^{\frac{1}{3}} + c$ **n** $y = -5x + c$
o $y = 3x^2 + c$ **p** $y = \frac{10}{3}x^{0.6} + c$

2 **a** $y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$ **b** $y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$
c $y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$ **d** $y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$
e $y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$ **f** $y = x^5 + 2x^{-\frac{3}{2}} + 3x^{-4} + c$

3 **a** $f(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$ **b** $f(x) = x^6 - x^{-6} + x^{\frac{1}{6}} + c$
c $f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$ **d** $f(x) = 2x^5 - 4x^{-2} + c$
e $f(x) = 3x^{\frac{2}{3}} - 6x^{-\frac{2}{3}} + c$
f $f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{3}} + c$

4 $y = \frac{4x^3}{3} + 6x^2 + 9x + c$

5 $f(x) = -3x^{-1} + 4x^{\frac{3}{2}} + \frac{x^2}{2} - 4x + c$

Sfidë

$y = -\frac{12}{7x^{\frac{7}{2}}} - \frac{4}{5x^{\frac{5}{2}}} + \frac{3}{2x^2} + \frac{1}{x} + c$

Ushtrime 13B

1 **a** $\frac{x^4}{4} + c$ **b** $\frac{x^8}{8} + c$
c $-x^{-3} + c$ **d** $\frac{5x^3}{3} + c$

2 **a** $\frac{1}{5}x^5 + \frac{1}{2}x^4 + c$ **b** $\frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$
c $2x^{\frac{5}{2}} - x^3 + c$

3 **a** $-4x^{-1} + 6x^{\frac{1}{2}} + c$ **b** $-6x^{-1} - \frac{2}{3}x^{\frac{3}{2}} + c$
c $-4x^{-\frac{1}{2}} + \frac{x^3}{3} - 2x^{\frac{1}{2}} + c$

4 **a** $x^4 + x^{-3} + rx + c$ **b** $\frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$
c $\frac{px^5}{5} + 2qx - 3x^{-1} + c$

5 **a** $t^3 + t^{-1} + c$ **b** $\frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$
c $\frac{p}{4}t^4 + q^2t + pr^3t + c$

6 **a** $x^2 - \frac{3}{x} + c$ **b** $\frac{4}{3}x^3 + 6x^2 + 9x + c$
c $\frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

7 **a** $\frac{1}{3}x^3 + 2x - \frac{1}{x} + c$ **b** $\frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$
c $2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

8 **a** $\frac{3}{5}x^{\frac{5}{3}} - \frac{2}{x^2} + c$ **b** $-\frac{1}{x^2} - \frac{1}{x} + 3x + c$
c $\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$ **d** $\frac{8}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$
e $3x + 2x^{\frac{1}{2}} + 2x^3 + c$ **f** $\frac{2}{5}x^{\frac{5}{2}} + 3x^2 + 6x^{\frac{3}{2}} + c$

9 **a** $-\frac{A}{x} - 3x + c$ **b** $\frac{2}{3}\sqrt{P}\,x^{\frac{3}{2}} - \frac{1}{x^2} + c$
c $-\frac{p}{x} + \frac{2qx^{\frac{3}{2}}}{3} + rx + c$

10 $-\frac{6}{x} + \frac{8x^{\frac{3}{2}}}{3} - \frac{3x^2}{2} + 2x + c$

11 $2x^4 + 3x^2 - 6x^{\frac{1}{2}} + c$

12 **a** $(2 + 5\sqrt{x})^2 = 4 + 10\sqrt{x} + 10\sqrt{x} + 25x = 4 + 20\sqrt{x} + 25x$
b $4x + \frac{40x^{\frac{3}{2}}}{3} + \frac{25x^2}{2} + c$

13 $\frac{x^6}{2} - 8x^{\frac{1}{2}} + c$

14 $p = -4$, $q = -2.5$

15 **a** $1024 - 5120x + 11\,520x^2$
b $1024x - 2560x^2 + 3840x^3 + c$

Ushtrime 13C

1 **a** $y = x^3 + x^2 - 2$ **b** $y = x^4 - \frac{1}{x^2} + 3x + 1$
c $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + \frac{1}{3}$ **d** $y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$
e $y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$ **f** $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$

2 $f(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$

3 $y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$

4 $f(x) = 3x^3 + 2x^2 - 3x - 2$

5 $y = 6x^{\frac{1}{2}} - \frac{4x^{\frac{5}{2}}}{5} + \frac{118}{5}$

6 **a** $p = \frac{1}{2}$, $q = 1$ **b** $y = 4x^{\frac{3}{2}} + \frac{5x^2}{2} - \frac{421}{2}$

7 **a** $f(t) = 10t - \frac{5t^2}{2}$ **b** $7\frac{1}{2}$

8 **a** $f(t) = -4.9t^2 + 35$ **b** 23.975 m
c 35 m **d** 2.67 sekonda
e p.sh. toka është e sheshtë.

Sfidë

1 $f_2(x) = \frac{x^3}{3}$; $f_4(x) = \frac{x^4}{12}$ **b** $\frac{x^{n+1}}{3 \times 4 \times 5 \times \ldots \times (n+1)}$

2 $f_2(x) = x + 1$; $f_3(x) = \frac{1}{2}x^2 + x + 1$; $f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

Ushtrime 13D

1 **a** $152\frac{1}{4}$ **b** $48\frac{2}{5}$ **c** $5\frac{1}{3}$ **d** 2
2 **a** $5\frac{1}{4}$ **b** 10 **c** $11\frac{5}{6}$ **d** $60\frac{1}{2}$
3 **a** $16\frac{2}{3}$ **b** $46\frac{1}{2}$ **c** $\frac{11}{14}$ **d** $2\frac{1}{2}$
4 $A = -7$ ose 4
5 28
6 $-8 + 8\sqrt{3}$
7 $k = \frac{25}{4}$
8 450 m

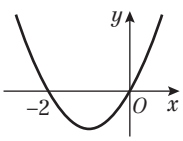
Sfidë
$k = 2$

Ushtrime 13E

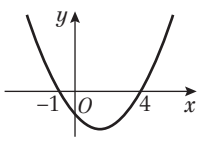
1 **a** 22 **b** $36\frac{2}{3}$ **c** $48\frac{8}{15}$ **d** 6
2 4 **3** 6 **4** $10\frac{2}{3}$
5 $21\frac{1}{3}$ **6** $\frac{4}{81}$ **7** $k = 2$
8 **a** (−1, 0) dhe (3, 0) **b** $10\frac{2}{3}$
9 $1\frac{1}{3}$

Ushtrime 13F

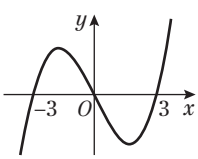
1 **a** $1\frac{1}{3}$



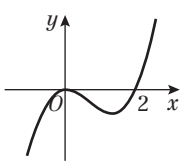
b $20\frac{5}{6}$



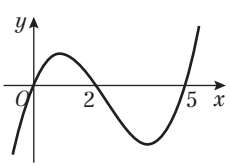
c $40\frac{1}{2}$



d $1\frac{1}{3}$



e $21\frac{1}{12}$



2 **a** (−3, 0) dhe (2, 0) **b** $21\frac{1}{12}$
3 **a** $f(-3) = 0$
b $f(x) = (x + 3)(-x^2 + 7x - 10)$
c $f(x) = (x + 3)(x - 5)(2 - x)$
d (−3, 0), (2, 0) dhe (5, 0)
e $143\frac{5}{6}$

Sfidë

1 **a** $4\frac{1}{2}$ **b** 9 **c** $\frac{9a}{2}$ **d** $4\frac{1}{2}$ **e** $\frac{9}{2a}$
2 **a** B e ka koordinatën x të barabartë me 1.
$\int_0^1 (x^3 + x^2 - 2x)dx = \left[\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right]_0^1$
$= \frac{1}{4} + \frac{1}{3} - 1 = -\frac{5}{12}$
Pra syprina nën boshtin x është $\frac{5}{12}$
syprina mbi x është
$\left(\frac{1}{4}0^4 + \frac{1}{3}0^3 - 0^2\right) - \left(\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right) = \frac{5}{12}$
Pra koordinata x e a kënaq
$3x^4 + 4x^3 - 12x^2 + 5 = 0$
Pastaj përdor teoremën faktor dy herë që të gjesh
$(x - 1)^2(3x^2 + 10x + 5) = 0$
b A ka koordinata $\left(\frac{-5 + \sqrt{10}}{3}, \frac{-80 + 37\sqrt{10}}{27}\right)$
Rrënjët në 1 i korrespondojnë pikës B.
Rrënja $\frac{-5 - \sqrt{10}}{3}$ jep një pikë në vijë në të majtë të -2
nën boshtin x, pra nuk mund të jetë A.

Ushtrime 13G

1 **a** $A(-2, 6)$, $B(2, 6)$ **b** $10\frac{2}{3}$
2 **a** $A(1, 3)$, $B(3, 3)$ **b** $1\frac{1}{3}$
3 $6\frac{2}{3}$
4 4.5
5 **a** (2, 12) **b** $13\frac{1}{3}$
6 **a** $20\frac{5}{6}$ **b** $17\frac{1}{6}$
7 **a, b** Zëvendëso në ekuacion për y
c $y = x - 4$ **d** $8\frac{3}{5}$
8 $3\frac{3}{8}$
9 **a** Zëvendëso $x = 4$ në gjithë ekuacionin
b 7.2
10 **a** $21\frac{1}{3}$ **b** $2\frac{5}{9}$
11 **a** (−1, 11) dhe (3, 7) **b** $21\frac{1}{3}$

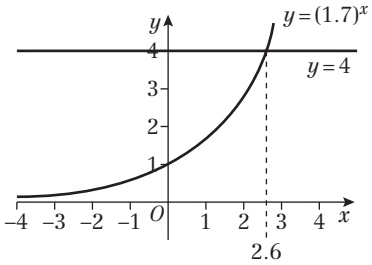
KAPITULLI 14

Kontrolli i njohurive të mëparshme

1 **a** 125 **b** $\frac{1}{3}$ **c** 32 **d** 49 **e** 1
2 **a** 6^6 **b** y^{21} **c** 2^6 **d** x^4
3 koeficienti këndor 1.5, ordinata në origjinë 4.1

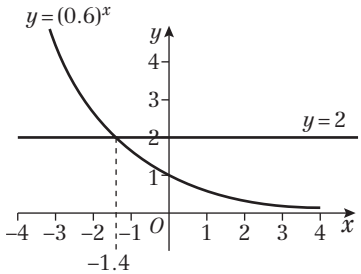
Ushtrime 14A

1 **a**



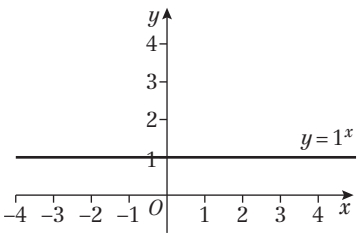
b $x \approx 2.6$

2 **a**



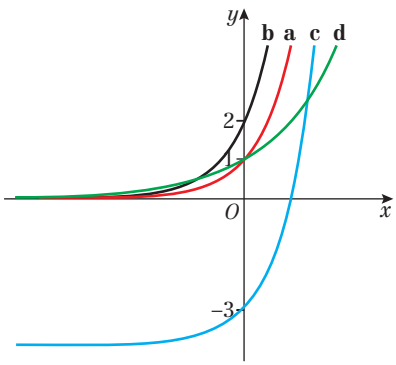
b $x \approx -1.4$

3



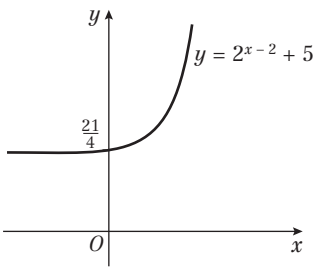
4 **a** I vërtetë, sepse $a^0 = 1$ për çdo a pozitive
b Jo i vërtetë, për shembull kur $a = \frac{1}{2}$
c I vërtetë, sepse kur a është pozitive, $a^x > 0$ për të gjitha vlerat e x

5



6 $k = 3$, $a = 2$
7 **a** Kur x rritet, y zvogëlohet
b $p = 1.2$, $q = 0.2$

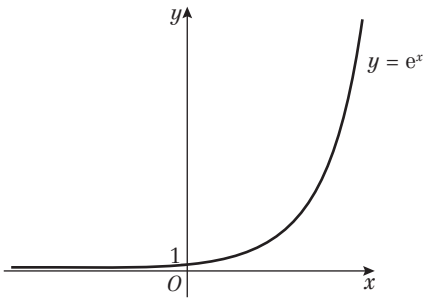
Sfidë



Ushtrime 14B

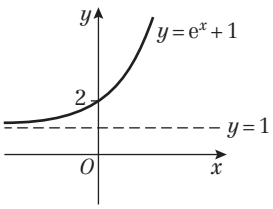
1 **a** 2.718 28 **b** 54.598 15 **c** 0.000 04 **d** 1.221 40

2 **a**

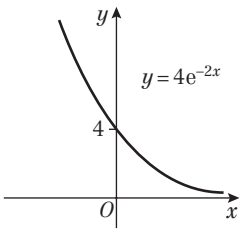


b Përgjigjet e nxënësit
c e = 2.71828…
$e^3 = 20.08553…$

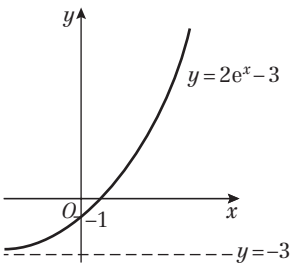
3 **a**



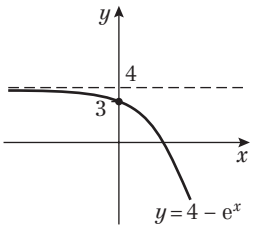
b



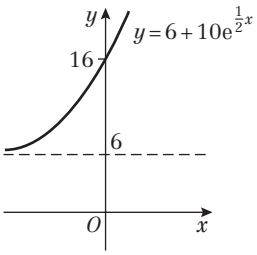
c



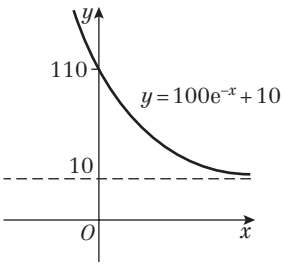
d



e

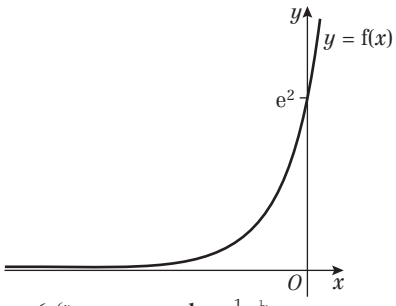


f



4 **a** $A = 1$, $C = 5$, b është pozitiv
b $A = 4$, $C = 0$, b është negativ
c $A = 6$, $C = 2$, b është pozitiv

5 $A = e^2$, $b = 3$



6 **a** $6e^{6x}$ **b** $-\frac{1}{3}e^{-\frac{1}{3}x}$ **c** $14e^{2x}$
d $2e^{0.4x}$ **e** $3e^{3x} + 2e^x$ **f** $2e^{2x} + e^x$

7 **a** $3e^6$ **b** 3 **c** $3e^{-1.5}$

8 $f'(x) = 0.2e^{0.2x}$
Koeficientin këndor i tangjentes kur $x = 5$ është
$f'(5) = 0.2e^1 = 0.2e$.
Ekuacioni i tangjentes rrjedhimisht është $y = (0.2e)x + c$.
Në (5, e), $e = 0.2e \times 5 + c$, pra $c = 0$ dhe kur $x = 0$, $y = 0$.

Ushtrime 14D

1 **a** $\log_4 256 = 4$ **b** $\log_3 \frac{1}{9} = -2$
c $\log_{10} 1\,000\,000 = 6$ **d** $\log_{11} 11 = 1$
e $\log_{0.2} 0.008 = 3$

2 **a** $2^4 = 16$ **b** $5^2 = 25$
c $9^{\frac{1}{2}} = 3$ **d** $5^{-1} = 0.2$
e $10^5 = 100\,000$

3 **a** 3 **b** 2 **c** 7 **d** 1
e 6 **f** $\frac{1}{2}$ **g** −1 **h** −2
i 10 **j** −2

4 **a** 625 **b** 9 **c** 7 **d** 9
e 20 **f** 2

5 **a** 2.475 **b** 2.173 **c** 3.009 **d** 1.099

6 **a** $5 = \log_2 32 < \log_2 50 < \log_2 64 = 6$
b 5.644

7 **a** **i** 1 **ii** 1 **iii** 1 **b** $a^1 \equiv a$

8 **a** **i** 0 **ii** 0 **iii** 0 **b** $a^0 \equiv 1$

Ushtrime 14E

1 **a** $\log_2 21$ **b** $\log_2 9$ **c** $\log_5 80$
d $\log_6 \left(\frac{64}{81}\right)$ **e** $\log_{10} 120$

2 **a** $\log_2 8 = 3$ **b** $\log_6 36 = 2$ **c** $\log_{12} 144 = 2$
d $\log_8 2 = \frac{1}{3}$ **e** $\log_{10} \frac{1}{10} = -1$

3 **a** $3\log_a x + 4\log_a y + \log_a z$
b $5\log_a x - 2\log_a y$
c $2 + 2\log_a x$
d $\log_a x - \frac{1}{2}\log_a y - \log_a z$
e $\frac{1}{2} + \frac{1}{2}\log_a x$

4 **a** $\frac{4}{3}$ **b** $\frac{1}{18}$ **c** $\sqrt{30}$ **d** 2

5 **a** $\log_3 (x + 1) - 2\log_3 (x - 1) = 1$

$$\log_3 \left(\frac{x+1}{(x-1)^2}\right) = 1$$

$$\frac{x+1}{(x-1)^2} = 3$$

$$x + 1 = 3(x-1)^2$$

$$x + 1 = 3(x^2 - 2x + 1)$$

$$3x^2 - 7x + 2 = 0$$

b $x = 2$

6 $a = 9$, $b = 4$

Sfidë
$\log_a x = m$ dhe $\log_a y = n$
$x = a^m$ dhe $y = a^n$
$x \div y = a^m \div a^n = a^{m-n}$

$$\log_a \left(\frac{x}{y}\right) = m - n = \log_a x - \log_a y$$

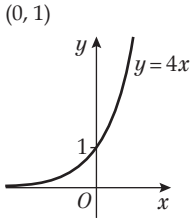
Ushtrime 14F

1 **a** 6.23 **b** 2.10 **c** 0.431
d 1.66 **e** −3.22 **f** 1.31
g 1.25 **h** −1.73

2 **a** 0, 2.32 **b** 1.26, 2.18 **c** 1.21
d 0.631 **e** 0.565, 0.712 **f** 0
g 2 **h** −1

3 **a** 5.92 **b** 3.2

4 **a** (0, 1)



b $\frac{1}{2}, \frac{3}{2}$

5 **a** 0.7565 **b** 7.9248 **c** 0.2966

Ushtrime 14G

1 **a** $\ln 6$ **b** $\frac{1}{2}\ln 11$ **c** $3 - \ln 20$
d $\frac{1}{4}\ln\left(\frac{1}{6}\right)$ **e** $\frac{1}{2}\ln 3 - 3$ **f** $5 - \ln 19$

2 **a** e^2 **b** $\frac{e}{4}$ **c** $\frac{1}{2}e^4 - \frac{3}{2}$
d $\frac{1}{6}(e^5 + 2)$ **e** $18 - e^{\frac{1}{2}}$ **f** $2, 5$

3 **a** $\ln 2, \ln 6$ **b** $\frac{1}{2}\ln 2, 0$ **c** e^3, e^{-5}
d $\ln 4, 0$ **e** $\ln 5, \ln\left(\frac{1}{6}\right)$ **f** e^6, e^{-2}

4 $\ln 3, 2\ln 2$

5 **a** $\frac{1}{8}(e^2 + 3)$ **b** $\frac{1}{5}(\ln 3 + 40)$ **c** $\frac{1}{5}\ln 7, 0$
d e^3, e^{-1}

6 $\dfrac{1 + \ln 5}{4 + \ln 3}$

7 **a** Përqendrimi i fillimit të mjekimit në mg/l
b 4.91 mg/l
c $3 = 6e^{-\frac{t}{10}}$
$\frac{1}{2} = e^{-\frac{t}{10}}$
$\ln\left(\frac{1}{2}\right) = -\dfrac{t}{10}$
$t = -10\ln\left(\frac{1}{2}\right) = 6.931\ldots = 6$ hours 56 minutes

8 **a** $(0, 3 + \ln 4)$ **b** $(4 - e^{-3})$

Sfidë
Me qenë se $y = 2$ është një asimptotë, $C = 2$.
Duke zëvendësuar $(0, 5)$, gjendet $5 = Ae^0 + 2$, pra A është 3.
Duke zëvendësuar $(6, 10)$ gjendet $10 = 3e^{6B} + 2$.
Duke rishkruar këtë, kemi $B = \frac{1}{6}\ln\left(\frac{8}{3}\right)$.

Mësimi 15.1

1 a Viti 1: 8, Viti 2: 12, Viti 3: 16.

b Çdo njëra nga: zgjedhja pasqyron strukturën e popullimit të shkollës me saktësi; garanton paraqitjen e përpjesshme të grupeve të ndryshme sipas viteve në zgjedhje.

2 a Zgjedhja mund të zbulojë ligjësi po qe se zgjidhet çdo person i njëzetë.

b Një zgjedhje e rastit e thjeshtë duke përdorur listën alfabetike si kuadër për zgjedhjen

3 a Jo: një zgjedhje sistematike kërkon që personi i parë të zgjidhet rastësisht.

b Bëni një zgjedhje të rastit të thjeshtë duke përdorur listën e anëtarëve si kuadër zgjedhjeje

4 a Zgjedhje e shtresëzuar (e stratifikuar)

b Mashkull Y12: 10, Mashkull Y13: 7, Femër Y12: 12, Femër Y13: 11.

5 a k=480/30 = 16

b Zgjidh rastësisht një numër midis 1 dhe 16. Duke u nisur nga punëtori me numër 16, zgjidh mbas kësaj çdo punëtor të 16-të.

6 a Çdo metodë në të cilën çdo individ i popullimit ka shanse të barabarta që të zgjidhet, p.sh një lotari. Disavantazhi: zgjedhja mund të mos paraqesë me siguri pjesët e anëtarëve të klubit që luajnë çdo sport.

b Zgjedhja do të ketë një përfaqësim të përpjesshëm të anëtarëve që luajnë sporte të ndryshme.

c Kriket: 10, Hokej: 12, Skuash: 8.

Mësimi 15.2

1 a Sasior b Cilësor
c Sasior d Sasior
e Cilësor

2 a Diskret b I vazhdueshëm
c Diskret c I vazhdueshëm
e I vazhdueshëm f I vazhdueshëm

3 a Është ndryshore cilësore sepse nuk është numër.

b Është sasior sepse është një numër. Është diskret sepse vlera e tij është një numër i

plotë; ju nuk mund të flisni për pjesë të një fëmije.

c Është sasior sepse është numër. Është i vazhdueshëm sepse mund të marrë çdo vlerë në një interval të dhënë.

4 a 1.4 kg dhe 1.5 kg b 1.35 kg c 0.1 kg

Mësimi 15.3

1 a 700 g b 600 g c 700 g

d Mesatarja do të rritet; moda nuk do të ndryshojë; mesorja do të zvogëlohet.

2 a 42.7

b Mesatarja do të rritet.

3 a Maj: 23 355 m, Qershor: 21 067 m

b 22 230 m

4 a 8 minuta b 10.2 minuta c 8.5 minuta

d Më mirë do të ishte mesorja. Mesatarja ndikohet nga vlera ekstreme 26.

5 a 2 b 1 c 1.47 d mesorja

6 6.31 petale

7 1

Mësimi 15.4

1 a 1020 hPa b Q_1 = 1017 hPa, Q_3 = 1024.5 hPa

2 Mesorja 37, Q_1 = 37, Q_3 = 38

3 1.08

4 a 432 kg b 389 kg c 480 kg

d Treçereku i lopëve peshon 480 kg ose më pak.

5 a 44.0 minuta b 48.8 minuta

c percentili 90-të = 57.8 minuta pra 10% e klientëve do të presin më shumë se 57.8 minuta, jo 56 minuta si thotë firma.

6 a 2.84 m. 80% e kondorëve kanë një hapje krahësh më pak se 2.84 m.

b Percentili 90-të është në klasën 3.0 £ ë. Nuk ka kufi të sipërm për këtë klasë, kështu që është e pamundur të vlerësohet percentili 90-të.

Mësimi 15.5

1 a 71 b 24.6 c 193.1 mm d 7

2 a £81.87 b 22

3 a 6.2 minuta b 54

4 a Mesorja 11.5 °C, Q_1 = 10.3 °C, Q_3 = 12.7 °C, IQR = 2.4 °C

b Mesatarisht, temperature ishte më e lartë në Qershor se në Maj (mesore më e madhe). Temperatura ishte më e ndryshueshme në Maj se në Qershor (IQR më e madhe).

c 24 ditë

Mësimi 15.6

1 Frekuenca

Masa, m (paund)

2 a Madhësia (koha) është e vazhdueshme.

b 150 c 369 d 699

3 a Madhësia (distanca, largësia) është e vazhdueshme.

b 310 c 75 d 95 e 65

4 a 32 qengja paraqiten nga 100 katrorë të vegjël, rrjedhimisht 25 katrorë të vegjël paraqesin 8 qengja.

b 32 c 68 d 88

5 a i

Koha , t (min) Denduria

ii

Densiteti i dendurisë

Koha, t (min)

b 35

6 a 12.5 dhe 14.5

b i 6 cm ii 3 cm

7 a Gjerësia 0.5 cm, lartësia 14 cm

b Mesatarja 10.4, mënjanimi mesatar katror 2.4

c 9 °C d 4.7 ditë

Mësimi 15.7

1 a Korrelacion pozitiv.

b Sa më i gjatë të jetë trajtimi, aq më e madhe është rënia e peshës.

2 a Nuk ka korrelacion.

b Reja e pikave nuk e mbështet pohimin se qytetet më të ngrohta kanë më pak reshje shiu.

3 a Masa para prekjes (g)

Masa mbasi u mbajt në dorë (g)

b Korrelacioni është pozitiv. Në qoftë se një nxënës thotë një masë më të madhe para prekjes, atëherë ka shumë të ngjarë që ai të thotë një masë më të madhe mbas e merr në dorë.

4 a Vlera e shtëpisë (mijë Euro)

Koha (s)

b Korrelacion i dobët negativ.

c Për shembull, mund të jetë një ndryshore e tretë që ndikon të dyja, vlerën e shtëpisë dhe lidhjen në internet, si largësia nga zonat e banuara.

5 a $Q_3 + 1.5 \times IQR = 4.85 + 7.125 = 11.975$

21.7 > 11.975, pra është një vlerë e veçantë.

b i Nuk ka arsye për të besuar se të dhënat e mbledhura janë jokorrekte.

ii 21.7 është një vlerë e veçantë, pra mund të mos jetë përfaqësuese e një sasie reshjesh tipike.

c s (orë)

r (mm)

d Korrelacion negative i dobët.

e Për shembull, mund të ketë një marrëdhënie shkakësore sepse ditët me më shumë reshje do të kenë më shumë re, dhe rrjedhimisht më pak diell.

Mësimi 15.8

1 a Kjo nuk është një ndryshore rasti diskrete sepse lartësia është një madhësi e vazhdueshme.

b Kjo është një ndryshore rasti diskrete, sepse numri i herëve është një numër i plotë dhe ai mund të jetë secili nga numrat 1, 2, 3, 4, 5 ose 6.

c Kjo nuk është një ndryshore rasti diskrete, sepse numri i ditëve në një javë të caktuar është përherë 7.

2 0, 1, 2, 3, 4

3 a (2, 2) (2, 3) (3, 2) (3, 3)

b

I	**x**		**4**	**5**	**6**
P(X = x)	**0.25**	**0.5**	**0.25**		

Ii P(X = x) =0.25, x = 4, 6 =0.5, x = 5

4 1/12

5 k + 2k + 3k + 4k = 1, pra 10k = 1, pra k = 1/10 .

6 a 0.125 b 0.875

13 a Shuma e probabiliteteve nuk është 1.

b 2 22/61

Sfidë

0.625

Mësimi 15.9

1 a, b

Kostot e prodhimit, p (në mijë$)

Numri i pjesëve, n (në mijë)

c Në qoftë se numri i pjesëve të prodhuara për muaj është zero, kostot e prodhimit do të jenë afërsisht 21 000$. Në qoftë se numri i pjesëve për muaj rritet me 1000 pjesë atëherë kostot e prodhimit rriten afërsisht me 980$.

d Parashikimi për 74 000 është brenda intervalit të të dhënave (interpolimi), rrjedhimisht me shumë të ngjarë është i sigurt. Parashikimi për 95 000 është jashtë intervalit të të dhënave (ekstrapolim), rrjedhimisht me pak të ngjarë është i sigurtë.

2 a Mbrojtja (vite)

Shtresa e bojës

b Një koeficient këndor I barabartë me 1.45 tregon se për do shtresë bojatisje shtesë, mbrojtja do të rritet me 1.45 vite, rrjedhimisht në qoftë se bëhen 10 shtresa me bojë, mbrojtja do të jetë 14.5 vite më gjatë se kur nuk bojatiset. Mbas 10 shtresave me bojë, mbrojtja do të zgjasë 2.93 + 14.5 = 17.43 vite.

3 a Numri i vezëve, n

Mosha, a (muaj)

b Reja e pikëve tregon se korrelacioni është negativ, rrjedhimisht koeficienti këndor në ekuacionin e regresit duhet të jetë negativ.

4 Kjo nuk është e arsyeshme sepse ka pak të ngjarë që të ketë shtëpi pa dhoma gjumi.

5 a Për çdo përqindje rritje të lagështisë maksimale relative ditore, shikueshmëria mesatare ditore zvogëlohet me 106 dm.

b Nivelet e larta të lagështisë relative mund të sjellin brymë ose mjegull dhe kjo e ul shikueshmërinë. Rrjedhimisht ka shumë të ngjarë që të ketë një lidhje shkakësore.

c i Parashikimi për 100% është jashtë intervalit të të dhënave, pra ka pak të ngjarë të jetë korrekt.

ii Ekuacioni i regresionit duhet të përdoret për të parashikuar një vlerë të v kur njihet h.

d Të dhënat janë të dobishme për të analizuar vetëm dy javët e para të Shtatorit. Vlerat e rastit gjatë Shtatorit mund të përdoren për të analizuar të gjithë muajin. Madhësia e zgjedhjes duhet të rritet me të dhëna të tjera shtesë që lidhen me muajt e tjerë.

Mësimi 15.10

1	a	0.9804	b	0.7382	c	0.5638	d	0.3020
2	a	0.9468	b	0.5834	c	0.1272	d	0.5989
3	a	0.5888	b	0.7662	c	0.1442	d	0.2302
4	a	0.8882	b	0.7992	c	0.0599	d	0.1258
5	a	0.0039	b	0.9648	c	0.3633		
6	a	0.2252	b	0.4613	c	0.7073		
7	a	k = 13	b	r = 28				
8	a	k = 1	b	r = 9	c	0.9801		

9 a X ~ B (10, 0.30) Supozime: Ndryshorja e rastit mund të marrë dy vlera (dëgjon ose nuk dëgjon), bëhet një numër i caktuar provash (10) dhe probabiliteti I çdo suksesi është I caktuar (0.3), çdo individ është zzgjedhur pavarësisht nga të tjerët në zgjedhje. pavarur.

b 0.1503 c s 5 8

10 a 0.2794 b 0.0378 c d = 5