

Përgjigjet

KAPITULLI 1

Kontrolli i njohurive të mëparshme

- 1 a $2m^2n + 3mn^2$ b $6x^2 - 12x - 10$
 2 a 2^8 b 2^4 c 2^6
 3 a $3x + 12$ b $10 - 15x$ c $12x - 30y$
 4 a 8 b $2x$ c xy
 5 a $2x$ b $10x$ c $\frac{5x}{3}$

Ushtrime 1A

- 1 a x^7 b $6x^5$ c k d $2p^2$
 e x f y^{10} g $5x^2$ h p^2
 i $2a^3$ j $2p$ k $6a^9$ l $3a^2b^3$
 m $27x^8$ n $24x^{11}$ o $63a^{12}$ p $32y^6$
 q $4a^6$ r $6a^{12}$
- 2 a $9x - 18$ b $x^2 + 9x$
 c $-12y + 9y^2$ d $xy + 5x$
 e $-3x^2 - 5x$ f $-20x^2 - 5x$
 g $4x^2 + 5x$ h $-15y + 6y^3$
 i $-10x^2 + 8x$ j $3x^3 - 5x^2$
 k $4x - 1$ l $2x - 4$
 m $9d^2 - 2c$ n $13 - r^2$
 o $3x^3 - 2x^2 + 5x$ p $14y^2 - 35y^3 + 21y^4$
 q $-10y^2 + 14y^3 - 6y^4$ r $4x + 10$
 s $11x - 6$ t $7x^2 - 3x + 7$
 u $-2x^2 + 26x$ v $-9x^3 + 23x^2$
- 3 a $3x^3 + 5x^5$ b $3x^4 - x^6$ c $\frac{x^3}{2} - x$
 d $4x^2 + \frac{5}{2}$ e $\frac{7x^6}{5} + x$ f $3x^4 - \frac{5x^2}{3}$

Ushtrime 1B

- 1 a $x^2 + 11x + 28$
 b $x^2 - x - 6$
 c $x^2 - 4x + 4$
 d $2x^2 + 3x - 2xy - 3y$
 e $4x^2 + 11xy - 3y^2$
 f $6x^2 - 10xy - 4y^2$
 g $2x^2 - 11x + 12$
 h $9x^2 + 12xy + 4y^2$
 i $4x^2 + 6x + 16xy + 24y$
 j $2x^2 + 3xy + 5x + 15y - 25$
 k $3x^2 - 4xy - 8x + 4y + 5$
 l $2x^2 + 5x - 7xy - 4y^2 - 20y$
 m $x^2 + 2x + 2xy + 6y - 3$
 n $2x^2 + 15x + 2xy + 12y + 18$
 o $13y - 4x + 12 - 4y^2 + xy$
 p $12xy - 4y^2 + 3y + 15x + 10$
 q $5xy - 20y - 2x^2 + 11x - 12$
 r $22y - 4y^2 - 5x + xy - 10$
- 2 a $5x^2 - 15x - 20$
 b $14x^2 + 7x - 70$
 c $3x^2 - 18x + 27$
 d $x^3 - xy^2$
 e $6x^3 + 8x^2 + 3x^2y + 4xy$
 f $x^2y - 4xy - 5y$
 g $12x^2y + 6xy - 8xy^2 - 4y^2$
 h $19xy - 35y - 2x^2y$
 i $10x^3 - 4x^2 + 5x^2y - 2xy$
 j $x^3 + 3x^2y - 2x^2 + 6xy - 8x$

- k $2x^2y + 9xy + xy^2 + 5y^2 - 5y$
 l $6x^2y + 4xy^2 + 2y^2 - 3xy - 3y$
 m $2x^3 + 2x^2y - 7x^2 + 3xy - 15x$
 n $24x^3 - 6x^2y - 26x^2 + 2xy + 6x$
 o $6x^3 + 15x^2 - 3x^2y - 18xy^2 - 30xy$
 p $x^3 + 6x^2 + 11x + 6$
 q $x^3 + x^2 - 14x - 24$
 r $x^3 - 3x^2 - 13x + 15$
 s $x^3 - 12x^2 + 47x - 60$
 t $2x^3 - x^2 - 5x - 2$
 u $6x^3 + 19x^2 + 11x - 6$
 v $18x^3 - 15x^2 - 4x + 4$
 w $x^3 - xy^2 - x^2 + y^2$
 x $8x^3 - 36x^2y + 54xy^2 - 27y^3$
- 3 $2x^2 - xy + 29x - 7y + 24$
 4 $4x^3 + 12x^2 + 5x - 6cm^3$
 5 $a = 12, b = 32, c = 3, d = -5$

Sfidë

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Ushtrime 1C

- 1 a $4(x + 2)$ b $6(x - 4)$
 c $5(4x + 3)$ d $2(x^2 + 2)$
 e $4(x^2 + 5)$ f $6x(x - 3)$
 g $x(x - 7)$ h $2x(x + 2)$
 i $x(3x - 1)$ j $2x(3x - 1)$
 k $5y(2y - 1)$ l $7x(5x - 4)$
 m $x(x + 2)$ n $y(3y + 2)$
 o $4x(x + 3)$ p $5y(y - 4)$
 q $3xy(3y + 4x)$ r $2ab(3 - b)$
 s $5x(x - 5y)$ t $4xy(3x + 2y)$
 u $5y(3 - 4z^2)$ v $6(2x^2 - 5)$
 w $xy(y - x)$ x $4y(3y - x)$
- 2 a $x(x + 4)$ b $2x(x + 3)$
 c $(x + 8)(x + 3)$ d $(x + 6)(x + 2)$
 e $(x + 8)(x - 5)$ f $(x - 6)(x - 2)$
 g $(x + 2)(x + 3)$ h $(x - 6)(x + 4)$
 i $(x - 5)(x + 2)$ j $(x + 5)(x - 4)$
 k $(2x + 1)(x + 2)$ l $(3x - 2)(x + 4)$
 m $(5x - 1)(x - 3)$ n $2(3x + 2)(x - 2)$
 o $(2x - 3)(x + 5)$ p $2(x^2 + 3)(x^2 + 4)$
 q $(x + 2)(x - 2)$ r $(x + 7)(x - 7)$
 s $(2x + 5)(2x - 5)$ t $(3x + 5y)(3x - 5y)$
 u $4(3x + 1)(3x - 1)$ v $2(x + 5)(x - 5)$
 w $2(3x - 2)(x - 1)$ x $3(5x - 1)(x + 3)$
- 3 a $x(x^2 + 2)$ b $x(x^2 - x + 1)$
 c $x(x^2 - 5)$ d $x(x + 3)(x - 3)$
 e $x(x - 4)(x + 3)$ f $x(x + 5)(x + 6)$
 g $x(x - 1)(x - 6)$ h $x(x + 8)(x - 8)$
 i $x(2x + 1)(x - 3)$ j $x(2x + 3)(x + 5)$
 k $x(x + 2)(x - 2)$ l $3x(x + 4)(x + 5)$
- 4 $(x^2 + y^2)(x + y)(x - y)$
 5 $x(3x + 5)(2x - 1)$

Sfidë

$$(x - 1)(x + 1)(2x + 3)(2x - 3)$$

Ushtrime 1D

- 1 a x^5 b x^{-2} c x^4 d x^3
 e x^5 f $12x^0 = 12$ g $3x^{\frac{1}{2}}$ h $5x$
 i $6x^{-1}$ j $x^{\frac{5}{6}}$ k $x^{\frac{12}{5}}$ l $x^{\frac{1}{6}}$

- 2 a 5 b 729 c 3 d $\frac{1}{16}$
 e $\frac{1}{3}$ f $\frac{-1}{125}$ g 1 h 216
 i $\frac{125}{64}$ j $\frac{9}{4}$ k $\frac{5}{6}$ l $\frac{64}{49}$
- 3 a $8x^5$ b $\frac{5}{x^2} - \frac{2}{x^3}$ c $5x^4$
 d $\frac{1}{x^2} + 4$ e $\frac{2}{x^3} + \frac{1}{x^2}$ f $\frac{8}{27}x^6$
 g $\frac{3}{x} - 5x^2$ h $\frac{1}{3x^2} + \frac{1}{5x}$
- 4 a 3 b $\frac{16}{\sqrt[3]{x}}$
- 5 a $\frac{x}{2}$ b $\frac{32}{x^6}$

Ushtrime 1E

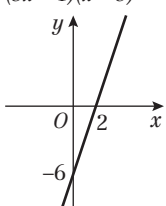
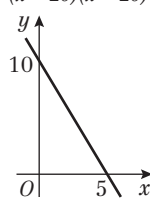
- 1 a $2\sqrt{7}$ b $6\sqrt{2}$ c $5\sqrt{2}$ d $4\sqrt{2}$
 e $3\sqrt{10}$ f $\sqrt{3}$ g $\sqrt{3}$ h $6\sqrt{5}$
 i $7\sqrt{2}$ j $12\sqrt{7}$ k $-3\sqrt{7}$ l $9\sqrt{5}$
 m $23\sqrt{5}$ n 2 o $19\sqrt{3}$
- 2 a $2\sqrt{3} + 3$ b $3\sqrt{5} - \sqrt{15}$
 c $4\sqrt{2} - \sqrt{10}$ d $6 + 2\sqrt{5} - 3\sqrt{2} - \sqrt{10}$
 e $6 - 2\sqrt{7} - 3\sqrt{3} + \sqrt{21}$ f $13 + 6\sqrt{5}$
 g $8 - 6\sqrt{3}$ h $5 - 2\sqrt{3}$
 i $3 + 5\sqrt{11}$
- 3 $3\sqrt{3}$

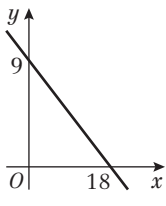
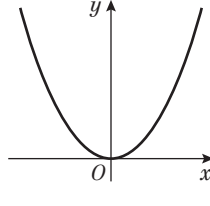
Ushtrime 1F

- 1 a $\frac{\sqrt{5}}{5}$ b $\frac{\sqrt{11}}{11}$ c $\frac{\sqrt{2}}{2}$
 d $\frac{\sqrt{5}}{5}$ e $\frac{1}{2}$ f $\frac{1}{4}$
 g $\frac{\sqrt{13}}{13}$ h $\frac{1}{3}$
- 2 a $\frac{1 - \sqrt{3}}{-2}$ b $\sqrt{5} - 2$ c $\frac{3 + \sqrt{7}}{2}$
 d $3 + \sqrt{5}$ e $\frac{\sqrt{5} + \sqrt{3}}{2}$ f $\frac{(3 - \sqrt{2})(4 + \sqrt{5})}{11}$
 g $5(\sqrt{5} - 2)$ h $5(4 + \sqrt{14})$ i $\frac{11(3 - \sqrt{11})}{-2}$
 j $\frac{5 - \sqrt{21}}{-2}$ k $\frac{14 - \sqrt{87}}{3}$ l $\frac{35 + \sqrt{189}}{6}$
 m -1
- 3 a $\frac{11 + 6\sqrt{2}}{49}$ b $9 - 4\sqrt{5}$ c $\frac{44 + 24\sqrt{2}}{49}$
 d $\frac{81 - 30\sqrt{2}}{529}$ e $\frac{13 + 2\sqrt{2}}{161}$ f $\frac{7 - 3\sqrt{3}}{11}$
- 4 $\frac{7 + \sqrt{5}}{4}$

KAPITULLI 2

Kontrolli i njohurive të mëparshme

- 1 a $x = -5$ b $x = 3$
 c $x = 5$ ose $x = -5$ d 16 ose 0
 2 a $(x + 3)(x + 5)$ b $(x + 5)(x - 2)$
 c $(3x + 1)(x - 5)$ d $(x - 20)(x + 20)$
- 3 a 
- b 

- 3 c 
- d 
- 4 a $x < 3$ b $x \neq 9$ c $x :: 2.5$ d $x > -7$

Ushtrime 2A

- 1 a $x = -1$ ose $x = -2$ b $x = -1$ ose $x = -4$
 c $x = -5$ ose $x = -2$ d $x = 3$ ose $x = -2$
 e $x = 3$ ose $x = 5$ f $x = 4$ ose $x = 5$
 g $x = 6$ ose $x = -1$ h $x = 6$ ose $x = -2$
 2 a $x = 0$ ose $x = 4$ b $x = 0$ ose $x = 25$
 c $x = 0$ ose $x = 2$ d $x = 0$ ose $x = 6$
 e $x = -\frac{1}{2}$ ose $x = -3$ f $x = -\frac{1}{3}$ ose $x = \frac{3}{2}$
 g $x = -\frac{2}{3}$ ose $x = \frac{3}{2}$ h $x = \frac{3}{2}$ ose $x = \frac{5}{2}$
 3 a $x = \frac{1}{3}$ ose $x = -2$ b $x = 3$ ose $x = 0$
 c $x = 13$ ose $x = 1$ d $x = 2$ ose $x = -2$
 e $x = \pm\sqrt{5}$ f $x = 3 \pm \sqrt{13}$
 g $x = \frac{1 \pm \sqrt{11}}{3}$ h $x = 1$ ose $x = -\frac{7}{6}$
 i $x = -\frac{1}{2}$ ose $x = \frac{7}{3}$ j $x = 0$ ose $x = -\frac{11}{6}$
- 4 $x = 4$
- 5 $x = -1$ ose $x = -\frac{2}{25}$

Ushtrime 2B

- 1 a $x = \frac{1}{2}(-3 \pm \sqrt{5})$ b $x = \frac{1}{2}(3 \pm \sqrt{17})$
 c $x = -3 \pm \sqrt{3}$ d $x = \frac{1}{2}(5 \pm \sqrt{33})$
 e $x = \frac{1}{3}(-5 \pm \sqrt{31})$ f $x = \frac{1}{2}(1 \pm \sqrt{2})$
 g $x = 2$ ose $x = -\frac{1}{4}$ h $x = \frac{1}{11}(-1 \pm \sqrt{78})$
- 2 a $x = -0.586$ ose $x = -3.41$ b $x = 7.87$ ose $x = 0.127$
 c $x = 0.765$ ose $x = -11.8$ d $x = 8.91$ ose $x = -1.91$
 e $x = 0.105$ ose $x = -1.90$ f $x = 3.84$ ose $x = -2.34$
 g $x = 4.77$ ose $x = 0.558$ h $x = 4.89$ ose $x = -1.23$
 3 a $x = -6$ ose $x = -2$ b $x = 1.09$ ose $x = -10.1$
 c $x = 9.11$ ose $x = -0.110$ d $x = -\frac{1}{2}$ ose $x = -2$
 e $x = 1$ ose $x = -9$ f $x = 1$
 g $x = 4.68$ ose $x = -1.18$ h $x = 3$ ose $x = 5$
- 4 Syprina = $\frac{1}{2}(2x)(x + (x + 10)) = 50m^2$
 Pra, $x^2 + 5x - 25 = 0$
 Nga formula kuadratike:
 $x = \frac{1}{2}(-5 \pm 5\sqrt{5})$



$$\text{Lartësia} = 2x = 5(\sqrt{5} - 1) \text{ m}$$

Sfidë

$$x = 13$$

Ushtrime 2C

- 1 a $(x+2)^2 - 4$
c $(x-8)^2 - 64$
e $(x-7)^2 - 49$
- 2 a $2(x+4)^2 - 32$
c $5(x+2)^2 - 20$
e $-2(x-2)^2 + 8$
- 3 a $2(x+2)^2 - 7$
c $3(x+\frac{1}{3})^2 - \frac{4}{3}$
e $-8(x-\frac{1}{8})^2 + \frac{81}{8}$
- 4 $a = \frac{3}{2}, b = \frac{15}{4}$
- 5 $A = 6, B = 0.04, C = -10$

Ushtrime 2D

- 1 a $x = -3 \pm 2\sqrt{2}$
c $x = -2 \pm \sqrt{6}$
- 2 a $x = \frac{1}{2}(-3 \pm \sqrt{15})$
c $x = \frac{1}{8}(1 \pm \sqrt{129})$
- 3 a $p = -7, q = -48$
b $(x-7)^2 = 48$
 $x = 7 \pm \sqrt{48} = 7 \pm 4\sqrt{3}$
 $r = 7, s = 4$
- 4 $x^2 + 2bx + c = (x+b)^2 - b^2 + c$
 $(x+b)^2 = b^2 - c$
 $x = -b \pm \sqrt{b^2 - c}$

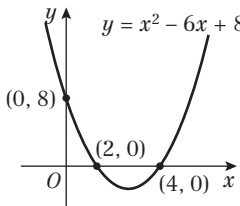
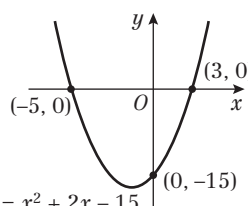
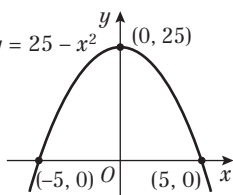
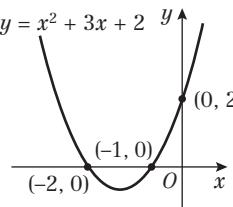
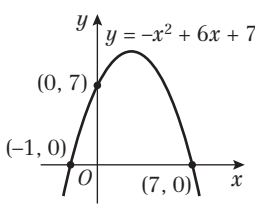
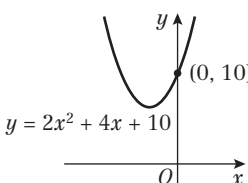
Sfidë

- a $ax^2 + 2bx + c = 0$
 $x^2 + \frac{2b}{a}x + \frac{c}{a} = 0$
 $(x + \frac{b}{a})^2 - \frac{b^2}{a^2} + \frac{c}{a} = 0$
 $(x + \frac{b}{a})^2 = \frac{b^2 - 4ac}{a^2}$
 $x = -\frac{b}{a} \pm \sqrt{\frac{b^2 - 4ac}{a^2}}$
- b $ax^2 + bx + c = 0$
 $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
 $(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$
 $(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$
 $x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$

Ushtrime 2E

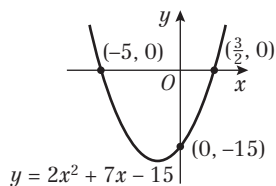
- 1 a 8 b 7 c 3 d 10.5 e 0
f 0 g 25 h 2 i 7
- 2 $a = 4$ ose $a = -2$
- 3 a $\frac{2}{3}$ b 2 dhe -9 c -10 dhe 4
d 12 dhe -12 e 0, -5 dhe -7 f 0, 3 dhe -8
- 4 $x = 3$ dhe $x = 2$
- 5 $x = 0, 2.5$ dhe 6
- 6 a $(x-1)^2 + 1$
 $p = -1, q = 1$
b Kufizat në katror janë përherë ≥ 0 , pra vlera minimale është $0 + 1 = 1$
- 7 a -2 dhe -1 b 2, -2, $2\sqrt{2}$ dhe $-2\sqrt{2}$
c -1 dhe $\frac{1}{3}$ d $\frac{1}{2}$ dhe 1
e 4 dhe 25 f 8 dhe -27
- 8 a $(3^x - 27)(3^x - 1)$ b 0 dhe 3

Ushtrime 2F

- 1 a 
 $y = x^2 - 6x + 8$
Pika e kthimit: (3, -1)
Drejtëza e simetrisë: $x = 3$
- b 
 $y = x^2 + 2x - 15$
Pika e kthimit: (-1, -16)
Drejtëza e simetrisë: $x = -1$
- c 
 $y = 25 - x^2$
Pika e kthimit: (0, 25)
Drejtëza e simetrisë: $x = 0$
- d 
 $y = x^2 + 3x + 2$
Pika e kthimit: $(-\frac{3}{2}, -\frac{1}{4})$
Drejtëza e simetrisë: $x = -\frac{3}{2}$
- e 
 $y = -x^2 + 6x + 7$
Pika e kthimit: (3, 16)
Drejtëza e simetrisë: $x = 3$
- f 
 $y = 2x^2 + 4x + 10$

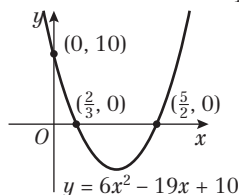
Pika e kthimit: $(-1, 8)$
Drejtëza e simetrisë: $x = -1$

g



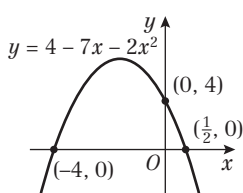
Pika e kthimit: $(-\frac{7}{4}, -\frac{169}{8})$
Drejtëza e simetrisë: $x = -\frac{7}{4}$

h



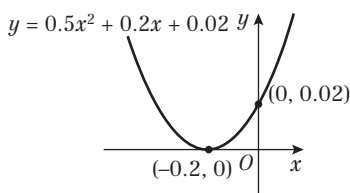
Pika e kthimit: $(\frac{19}{12}, -\frac{121}{24})$
Drejtëza e simetrisë: $x = \frac{19}{12}$

i



Pika e kthimit: $(-\frac{7}{4}, \frac{81}{8})$
Drejtëza e simetrisë: $x = -\frac{7}{4}$

j



Pika e kthimit: $(-0.2, 0)$
Drejtëza e simetrisë: $x = -0.2$

- 2 a $a = 1, b = -8, c = 15$
b $a = -1, b = 3, c = 10$
c $a = 2, b = 0, c = -18$
d $a = \frac{1}{4}, b = -\frac{3}{4}, c = -1$
- 3 a $a = 3, b = -30, c = 72$

Ushtrime 2G

- 1 a i 52 ii -23 iii 37
iv 0 v -44
b i $h(x)$ ii $f(x)$ iii $k(x)$
iv $j(x)$ v $g(x)$
- 2 $k < 9$
- 3 $t = \frac{9}{8}$
- 4 $s = 4$

- 5 $k > \frac{4}{3}$
- 6 a $p = 6$ b $x = -9$
- 7 a $k^2 + 16$
b k^2 është përherë pozitiv pra, $k^2 + 16 > 0$

Sfidë

- a Duhet $b^2 > 4ac$. Në qoftë se $a, c > 0$ ose $a, c < 0$, zgjidh b të tillë që $b > \sqrt{4ac}$. Në qoftë se $a > 0$ dhe $c < 0$ (ose anasjelltas), atëherë $4ac < 0$, pra $4ac < b^2$ për të gjitha b .
- b Jo në qoftë se njëra nga a ose c është negative sepse kjo do të kërkonte që b të ishte rrënja katrore e një numri negativ. E mundshme në qoftë se të dy janë negativë ose të dy janë pozitivë.

KAPITULLI 3

Kontrolli i njohurive të mëparshme

- 1 a $A \cap B = \{1, 2, 4\}$ b $(A \cup B)' = \{7, 9, 11, 13\}$
2 a $5\sqrt{3}$ b $\sqrt{5} + 2\sqrt{2}$
3 a graph ii b graph iii c graph i

Ushtrime 3A

- 1 a $x = 4, y = 2$ b $x = 1, y = 3$
c $x = 2, y = -2$ d $x = \frac{4}{2}, y = -3$
e $x = -\frac{2}{3}, y = 2$ f $x = 3, y = 3$
- 2 a $x = 5, y = 2$ b $x = 5\frac{1}{2}, y = -6$
c $x = 1, y = -4$ d $x = 1\frac{3}{4}, y = \frac{1}{4}$
- 3 a $x = -1, y = 1$ b $x = 4, y = -4$
c $x = 0.5, y = -2.5$
- 4 a $3x + ky = 8$ (1); $x - 2ky = 5$ (2)
(1) $\times 2$: $6x + 2ky = 16$ (3)
(2) + (3) $7x = 21$ pra $x = 3$
b -2
5 $p = 3, q = 1$

Ushtrime 3B

- 1 a $x = 5, y = 6$ ose $x = 6, y = 5$
b $x = 0, y = 1$ ose $x = \frac{4}{5}, y = -\frac{3}{5}$
c $x = -1, y = -3$ ose $x = 1, y = 3$
d $a = 1, b = 5$ ose $a = 3, b = -1$
e $u = \frac{1}{2}, v = 4$ ose $u = 2, v = 3$
f $x = -\frac{1}{2}, y = \frac{5}{4}$ ose $x = 3, y = -1$
- 2 a $x = 3, y = \frac{1}{2}$ ose $x = 6\frac{1}{3}, y = -\frac{25}{6}$
b $x = 4\frac{1}{2}, y = 4\frac{1}{2}$ ose $x = 6, y = 3$
c $x = -19, y = -15$ ose $x = 6, y = 5$
- 3 a $x = 3 + \sqrt{13}, y = -3 + \sqrt{13}$ ose $x = 3 - \sqrt{13}, y = -3 - \sqrt{13}$
b $x = 2 - 3\sqrt{5}, y = 3 + 2\sqrt{5}$ ose $x = 2 + 3\sqrt{5}, y = 3 - 2\sqrt{5}$
- 4 $x = -5, y = 8$ ose $x = 2, y = 1$
- 5 a $3x^2 + x(2 - 4x) + 11 = 0$
 $3x^2 + 2x - 4x^2 + 11 = 0$
 $x^2 - 2x - 11 = 0$
b $x = 1 + 2\sqrt{3}, y = -2 - 8\sqrt{3}$
 $x = 1 - 2\sqrt{3}, y = -2 + 8\sqrt{3}$
- 6 a $k = 3, p = -2$
b $x = -6, y = -23$

Sfidë

$$y = x + k$$

$$x^2 + (x + k)^2 = 4$$



$$x^2 + x^2 + 2kx + k^2 - 4 = 0$$

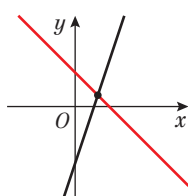
$$2x^2 + 2kx + k^2 - 4 = 0 \quad \text{for one solution } b^2 - 4ac = 0$$

$$4k^2 - 4 \times 2(k^2 - 4) = 0$$

$$4k^2 - 8k^2 + 32 = 0 \quad 4k^2 = 32 \quad k^2 = 8 \quad k = \pm 2\sqrt{2}$$

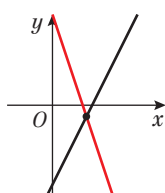
Ushtrime 3C

1 a i



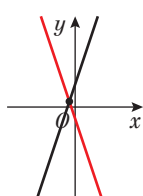
ii (2, 1)

b i



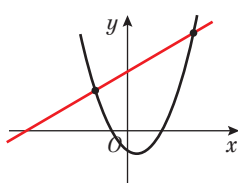
ii (3, -1)

c i



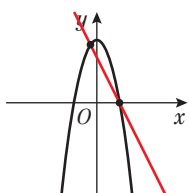
ii (-0.5, 0.5)

2 a



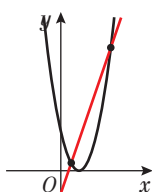
b (3.5, 9) dhe (-1.5, 4)

3 a



b (-1, 8) dhe (3, 0)

4 a



b (6, 16) dhe (1, 1)

5 (-11, -15) dhe (3, -1)

6 $(-\frac{1}{6}, -4\frac{1}{2})$ dhe (2, 5)

7 a 2 pikë

b 1 pikë

c 0 pikë

8 a $y = 2x - 1$

$$x^2 + 4k(2x - 1) + 5k = 0$$

$$x^2 + 8kx - 4k + 5k = 0 \quad x^2 + 8kx + k = 0$$

b $k = \frac{1}{16}$ c $x = -\frac{1}{4}, y = -\frac{3}{2}$

9 Një qoftë se notari arrin fundin e pishinës, atëherë

$$0.5x^2 - 3x = 0.3x - 6$$

$$0.5x^2 - 3.3x + 6 = 0$$

$$b^2 - 4ac = (-3.3)^2 - 4 \times 0.5 \times 6 = -1.11$$

negative, pra nuk ka pika prerje dhe kur zhytet ai nuk e arrin fundin e pishinës.

Ushtrime 3D

1 a $x < 4$ b $x \perp 7$ c $x > \frac{7}{2}$ d $x \leq -3$

e $x < 11$ f $x < \frac{23}{5}$ g $x > -12$ h $x < 1$

i $x \leq 8$ j $x > \frac{1}{7}$

2 a $x \geq 3$ b $x < 1$ c $x \leq -3\frac{1}{4}$ d $x < 18$

e $x > 3$ f $x \geq 4\frac{2}{5}$ g $x < 4$ h $x > -7$

i $x \leq -\frac{1}{2}$ j $x \geq \frac{3}{4}$ k $x \geq -\frac{10}{3}$ l $x \geq \frac{9}{11}$

3 a $\{x: x > \frac{21}{2}\}$ b $\{x: 2 < x < 4\}$

c $\{x: \frac{21}{2} < x < 3\}$ d Nuk ka vlera

e $x = 4$ f $\{x: x < 1.2\} \cup \{x: x > 2.2\}$

g $\{x: x \leq -\frac{2}{3}\} \cup \{x: x \geq \frac{3}{2}\}$

Sfidë

$$p = -1, q = 4, r = 6$$

Ushtrime 3E

1 a $3 < x < 8$ b $-4 < x < 3$

c $x < -2, x > 5$ d $x :: -4, x \perp -3$

e $-\frac{1}{2} < x < 7$ f $x < -2, x > \frac{1}{2}$

g $\frac{1}{2} \leq x \leq \frac{1}{2}$ h $x < \frac{1}{3}, x > 2$

i $-3 < x < 3$ j $x < -\frac{1}{2}, x > \frac{2}{3}$

k $x < 0, x > 5$ l $-\frac{1}{2} :: x :: 0$

2 a $-5 < x < 2$ b $x < -1, x > 1$

c $\frac{1}{2} < x < 1$ d $-3 < x < \frac{1}{4}$

3 a $\{x: 2 < x < 4\}$ b $\{x: x > 3\}$

c $\{x: -\frac{1}{4} < x < 0\}$ d Nuk ka vlera

e $\{x: -5 < x < -3\} \cup \{x: x > 4\}$

f $\{x: -1 < x < 1\} \cup \{x: 2 < x < 3\}$

4 a $x < 0$ ose $x > 2$ b $x < 0$ ose $x > 0.8$

c $x < -1$ ose $x > 0$ d $x < 0$ ose $x > 0.5$

e $x < -\frac{1}{5}$ ose $x > \frac{1}{5}$ f $x \leq -\frac{2}{3}$ ose $x \geq 3$

5 a $-2 < k < 6$ b $p \leq -8$ ose $p \geq 0$

6 $\{x: x < -2\} \cup \{x: x > 7\}$ b $\{x: -\frac{1}{2} < x < 3\}$

7 a $\{x: x < \frac{2}{3}\}$ b $\{x: -\frac{1}{2} < x < 3\}$

c $\{x: -\frac{1}{2} < x < \frac{2}{3}\}$

8 $x < 3$ ose $x > 5.5$

9 Nuk ka rrënjë reale kur $b^2 - 4ac < 0$ $(-2k)^2 - 4 \times k \times 3 < 0$

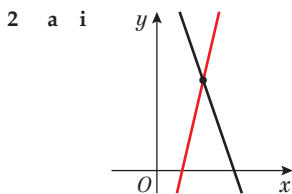
$$4k^2 - 12k = 0 \text{ kur } k = 0 \text{ dhe } k = 3$$

$$\text{zgjidhja } 0 \leq k < 3$$

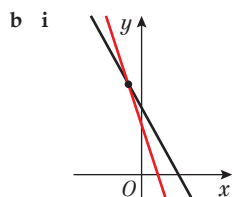
$$\text{vini re se kur } k = 0 \text{ ekuacioni jep } 3 = 0$$

Ushtrime 3F

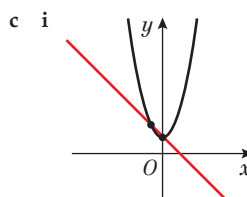
1 a $P(3.2, -1.8)$



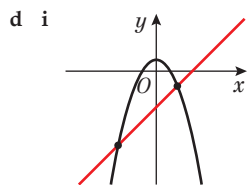
ii $(4, 5)$



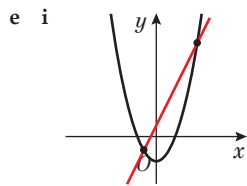
ii $(-3, 23)$



ii $(-2, 9), (0, 5)$



ii $(-5, -22), (3, -6)$



ii $(-2, -1), (9, 76)$

b $x < 3.2$

iii $x \leq 4$

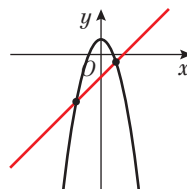
iii $x \geq -3$

iii $-2 \leq x \leq 0$

iii $x \leq -5$ ose $x \geq 3$

iii $-2 \leq x \leq 9$

f i



ii $(-5, -18), (3, -2)$

3 a $-1 < x < 2$

c $x < 0.5$ ose $x > 3$

e $1 < x < 3$

iii $x \leq -5$ ose $x \geq 3$

b $0.5 < x < 3$

d $x < 0$ ose $x > 2$

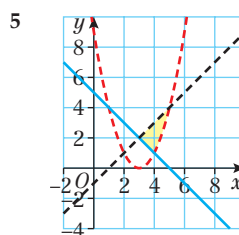
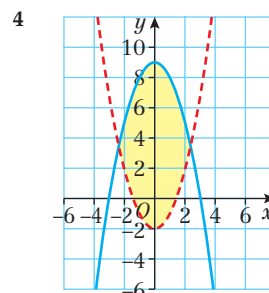
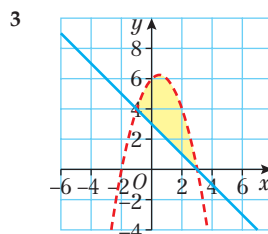
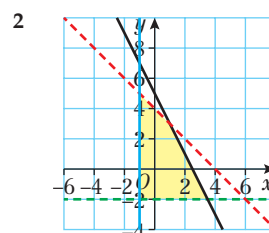
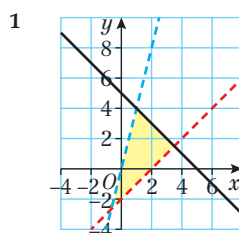
f $x < -1$ ose $x > -0.75$

Sfidë

a $(-1.5, -3.75), (6, 0)$

b $\{x: -1.5 < x < 6\}$

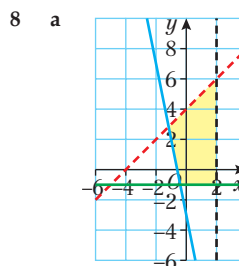
Ushtrime 3G



6 a $(1, 6), (3, 4), (1, 2)$

b $x \geq 1, y \leq 7 - x, y \geq x + 1$

7 $y < 2 - 5x - x^2, 2x + y \geq 0, x + y \leq 4$



b $(-\frac{7}{6}, \frac{17}{6}), (2, 6), (2, -1), (-0.4, -1)$

c $(-0.4, -1)$

d $\frac{941}{60}$

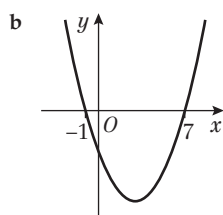
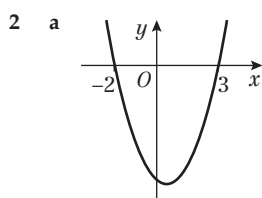


KAPITULLI 4

Kontrolli i njohurive të mëparshme

1 a $(x+5)(x+1)$

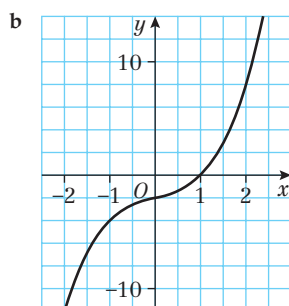
b $(x-3)(x-1)$



3 a

x	-2	-1.5	-1	-0.5	0
y	-12	-6.875	-4	-2.625	-2

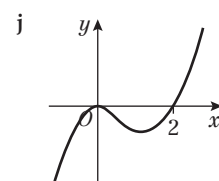
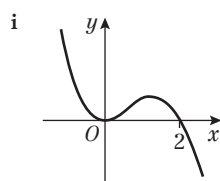
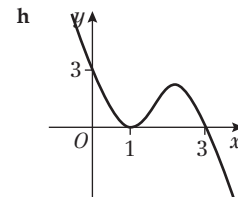
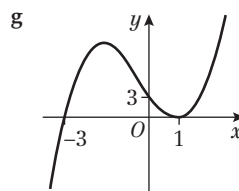
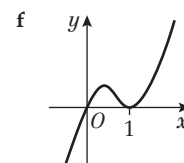
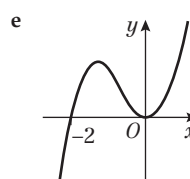
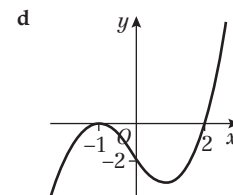
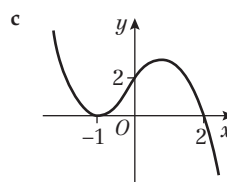
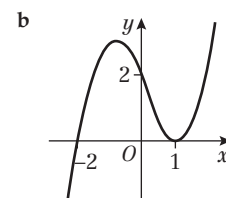
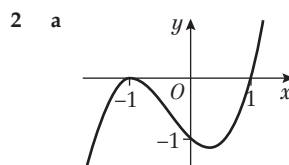
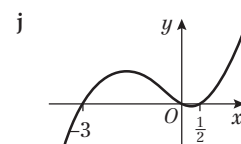
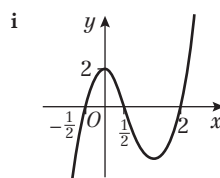
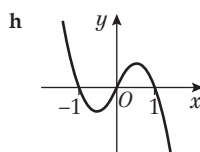
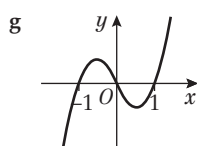
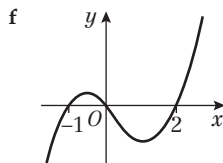
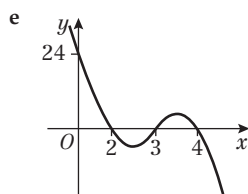
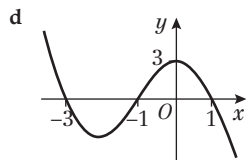
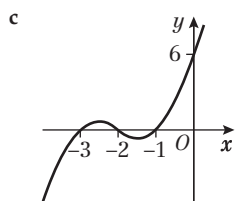
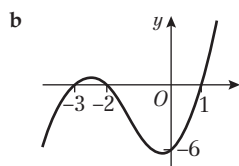
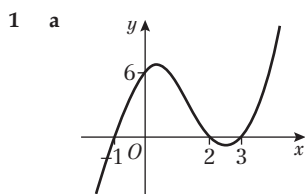
x	0.5	1	1.5	2
y	-1.375	0	2.875	8



4 a $x=2, y=4$

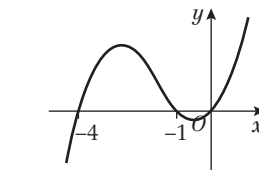
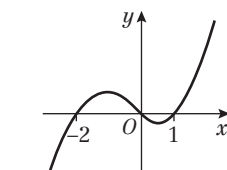
b $x=1, y=1$

Ushtrime 4A



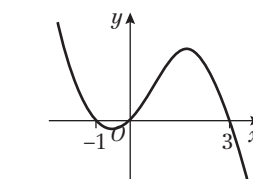
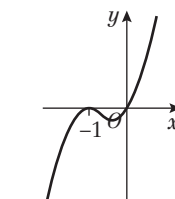
3 a $y=x(x+2)(x-1)$

b $y=x(x+4)(x+1)$

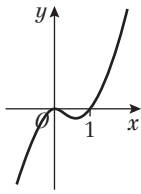


c $y=x(x+1)^2$

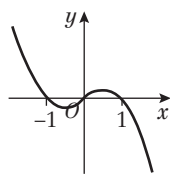
d $y=x(x+1)(3-x)$



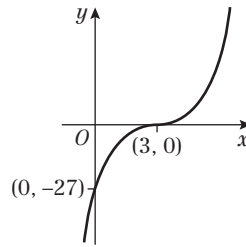
e $y = x^2(x - 1)$



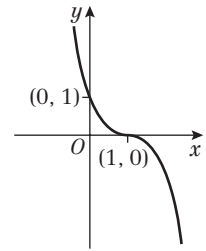
f $y = x(1 - x)(1 + x)$



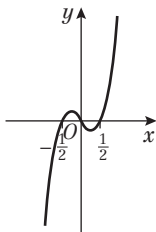
g



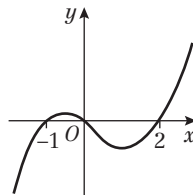
h



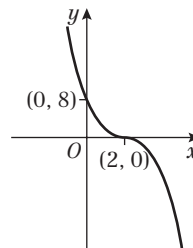
g $y = 3x(2x - 1)(2x + 1)$



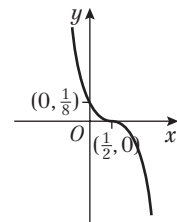
h $y = x(x + 1)(x - 2)$



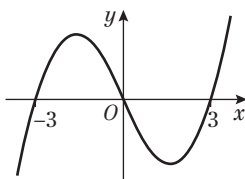
i



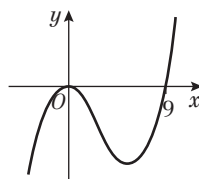
j



i $y = x(x - 3)(x + 3)$



j $y = x^2(x - 9)$

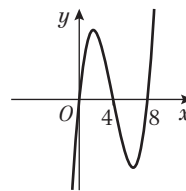


5 a $b = 4, c = 1, d = -6$

6 $a = \frac{1}{3}, b = -\frac{4}{3}, c = \frac{1}{3}, d = 2$

7 a $x(x^2 - 12x + 32)$

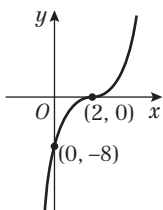
c



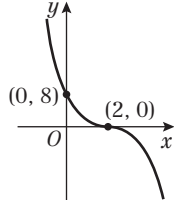
b $(0, -6)$

b $x(x - 8)(x - 4)$

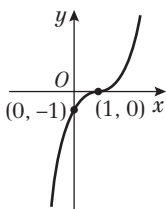
4 a



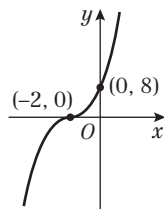
b



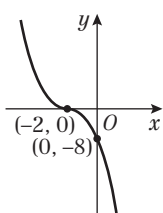
c



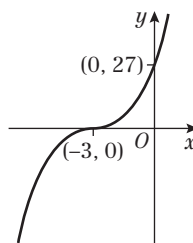
d



e

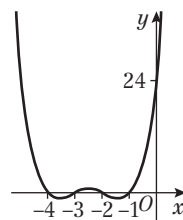


f

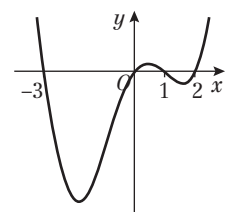


Ushtrime 4B

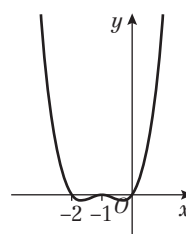
1 a



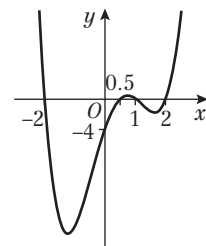
b



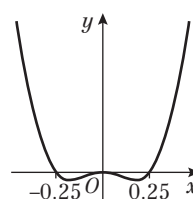
c



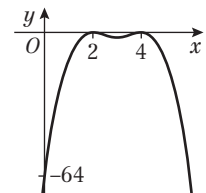
d

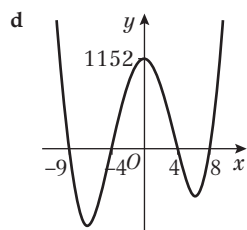
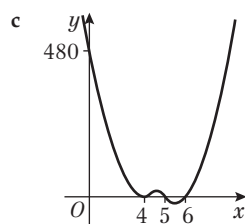
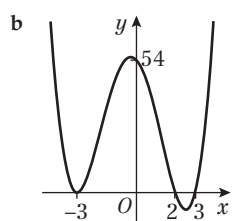
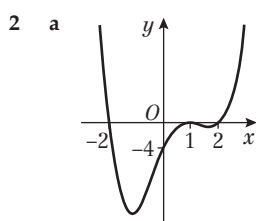
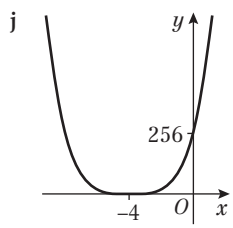
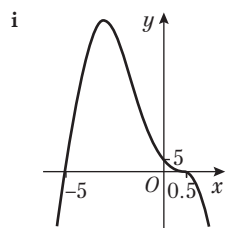
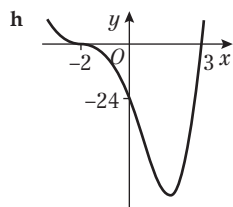
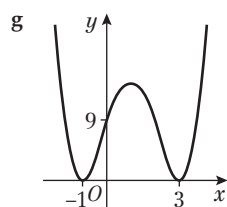


e

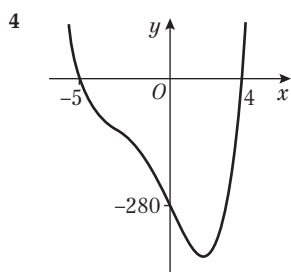


f





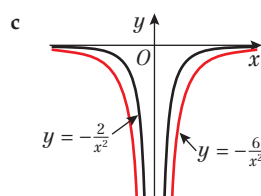
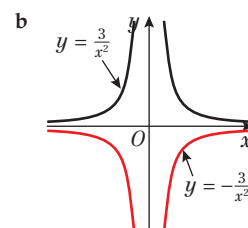
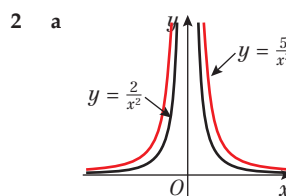
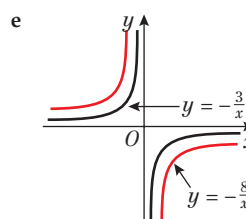
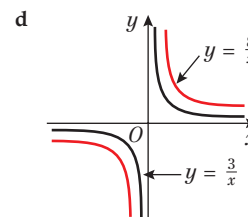
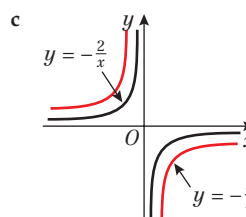
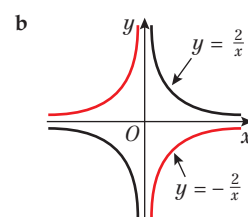
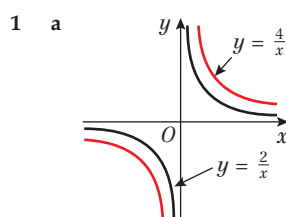
- 3 a $(0, 12)$
b $b = -2, c = -7, d = 8, e = 12$



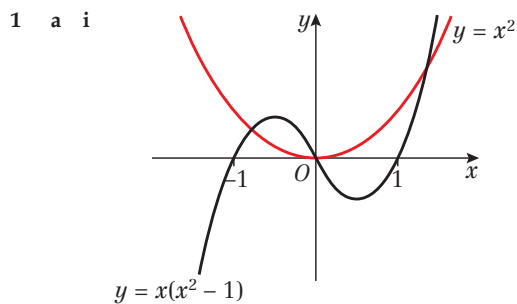
Sfidë

$$a = \frac{1}{3}, b = -\frac{4}{3}, c = -\frac{2}{3}, d = 4, e = 3$$

Ushtrime 4C



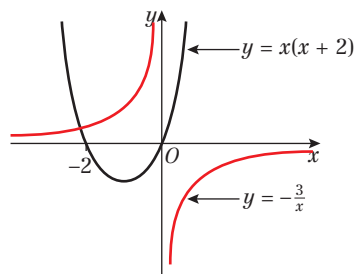
Ushtrime 4D



ii 3

iii $x^2 = x(x^2 - 1)$

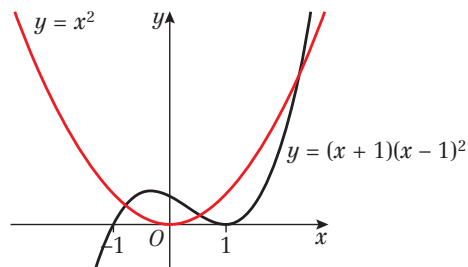
b i



ii 1

iii $x(x+2) = -\frac{3}{x}$

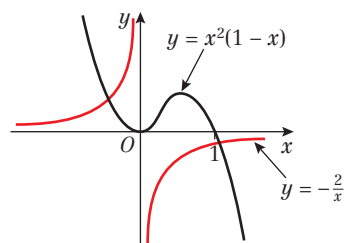
c i



ii 3

iii $x^2 = (x+1)(x-1)^2$

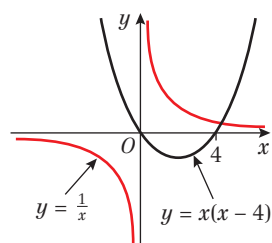
d i



ii 2

iii $x^2(1-x) = \frac{2}{x}$

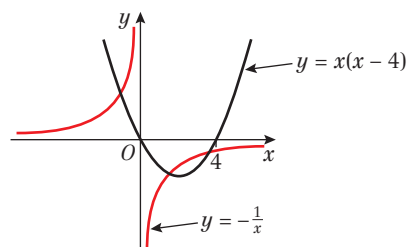
e i



ii 1

iii $x(x-4) = \frac{1}{x}$

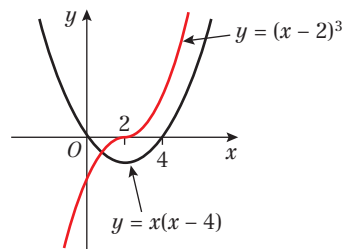
f i



ii 3

iii $x(x-4) = -\frac{1}{x}$

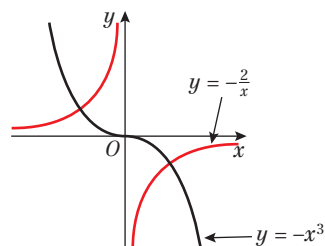
g i



ii 1

iii $x(x-4) = (x-2)^3$

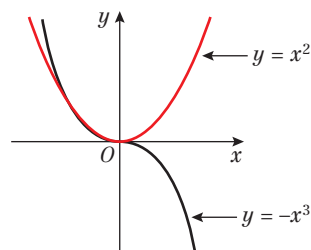
h i



ii 2

iii $-x^3 = -\frac{2}{x}$

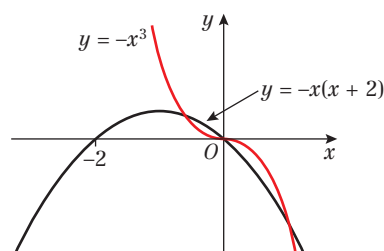
i i



ii 2

iii $-x^3 = x^2$

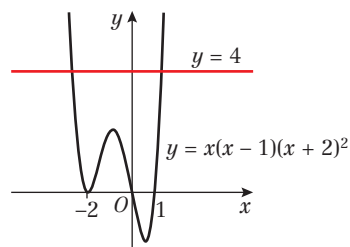
j i



ii 3

iii $-x^3 = -x(x+2)$

k i

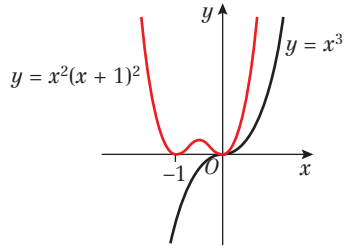


ii 2

iii $x(x-1)(x+2)^2 = 4$



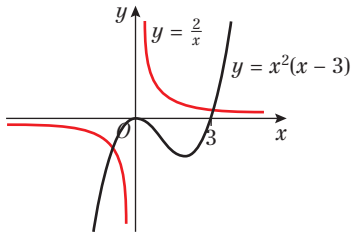
1 i



ii 1

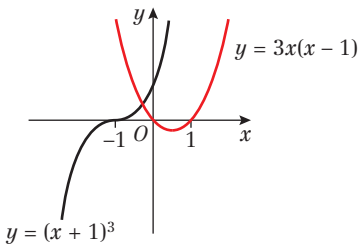
iii $x^3 = x^2(x+1)^2$

2 a



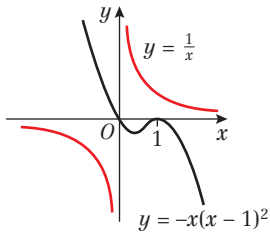
b Vetëm 2 pika prerje

3 a



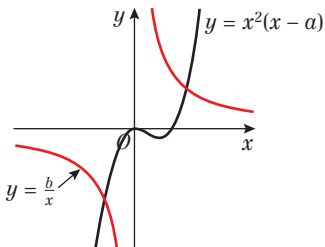
b Vetëm 1 pikëprerje

4 a



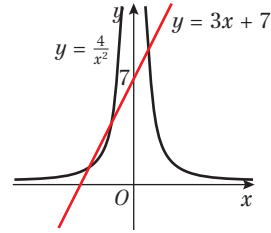
b Grafikët nuk priten

5 a



b 2: grafikët priten në dy vende, pra ka dy zgjidhje.

6 a

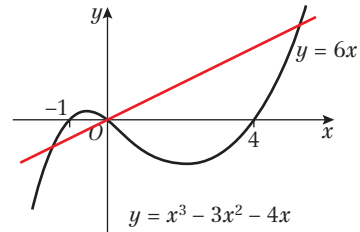


b 3

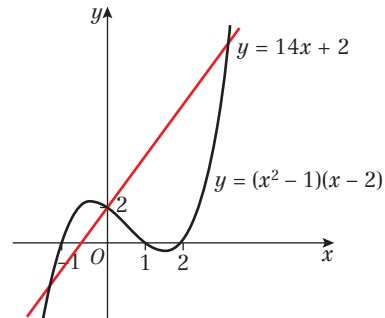
c Hap kllapat dhe thjeshto

d $(-2, 1), (-1, 4), (\frac{2}{3}, 9)$

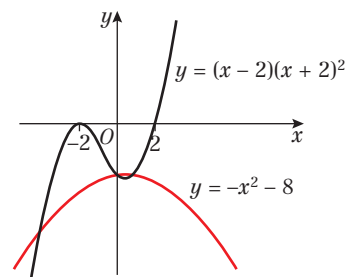
7 a

b $(0, 0); (-2, -12); (5, 30)$

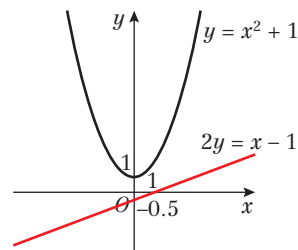
8 a

b $(0, 2); (-3, -40); (5, 72)$

9 a

b $(0, -8); (1, -9); (-4, -24)$

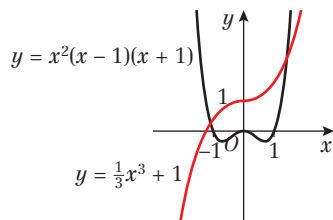
10 a



b Grafikët nuk priten.

c $a < -\frac{7}{16}$

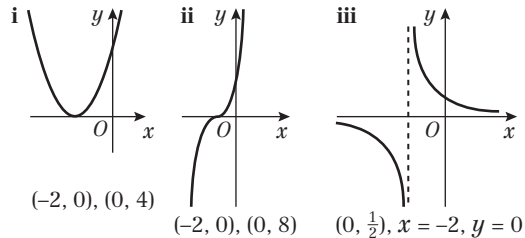
11 a



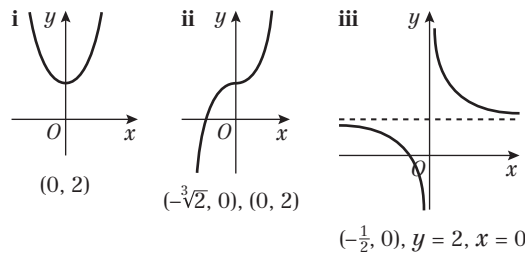
b 2

Ushtrime 4E

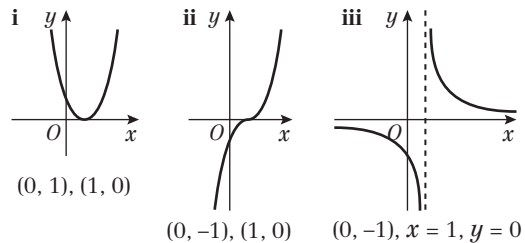
1 a



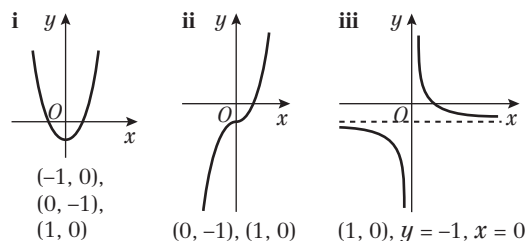
b



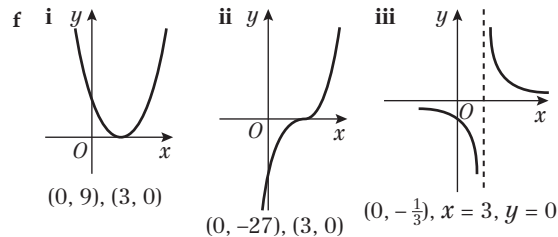
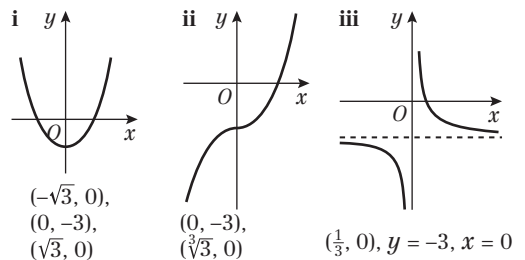
c



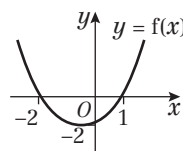
d



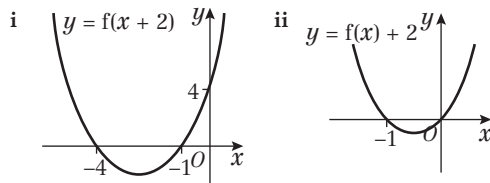
e



2 a



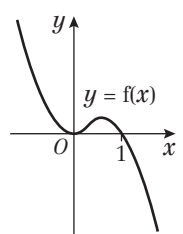
b



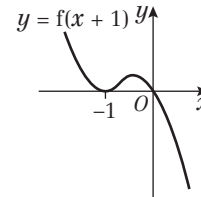
c

$f(x+2) = (x+1)(x+4); (0, 4)$
 $f(x) + 2 = (x-1)(x+2) + 2; (0, 0)$

3 a



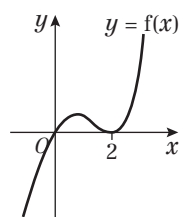
b



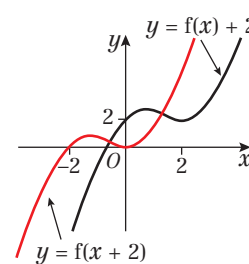
c

$f(x+1) = -x(x+1)^2; (0, 0)$

4 a



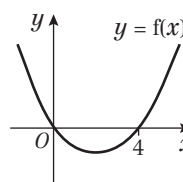
b



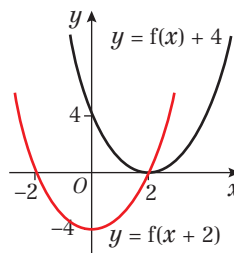
c

$f(x+2) = (x+2)x^2; (0, 0); (-2, 0)$

5 a



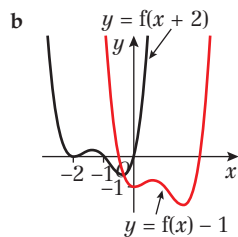
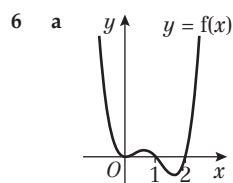
b



c

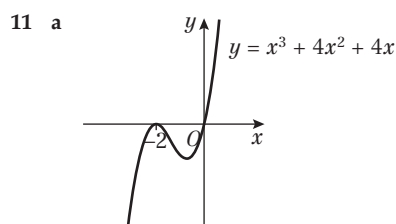
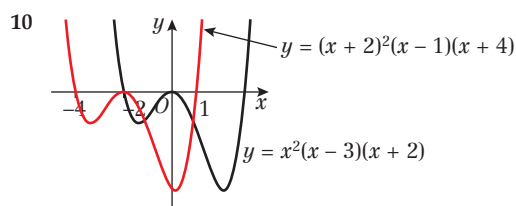
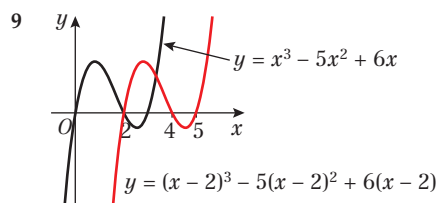
$f(x+2) = (x+2)(x-2); (2, 0); (-2, 0); (0, -4)$
 $f(x) + 4 = (x-2)^2; (2, 0); (0, 4)$



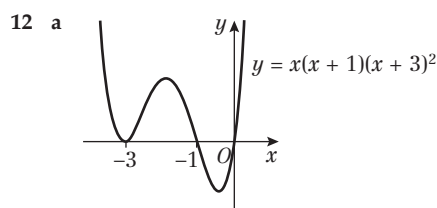


7 a $(6, -1)$ b $(4, 2)$

8 $y = \frac{1}{x-4}$



b -1 ose 1



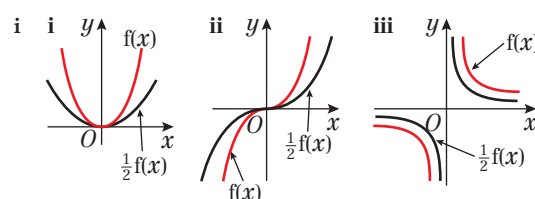
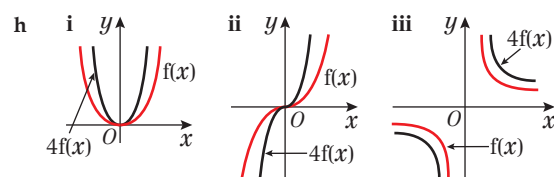
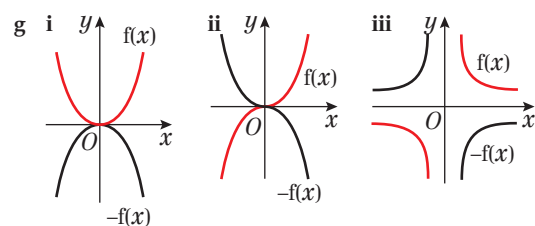
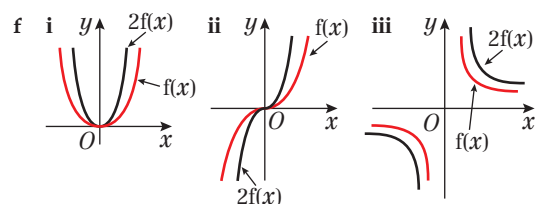
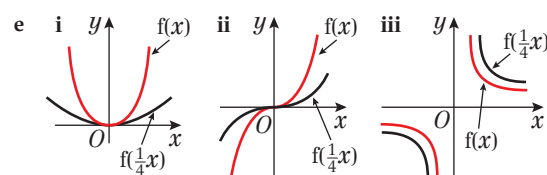
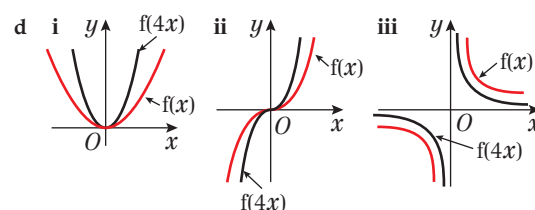
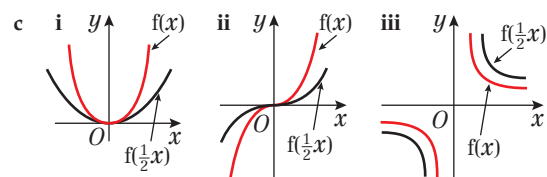
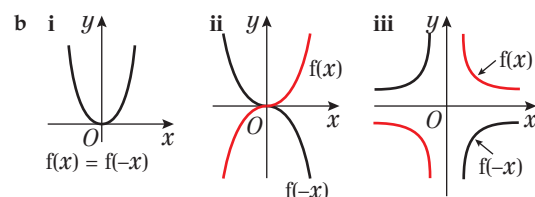
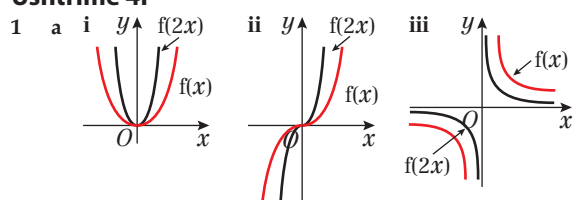
b $-2, -3$ ose -5

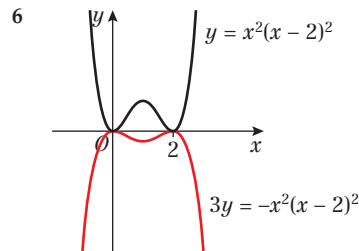
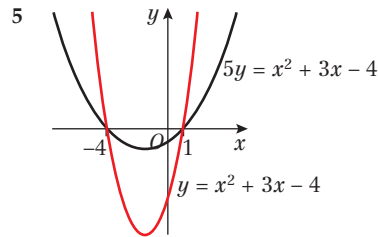
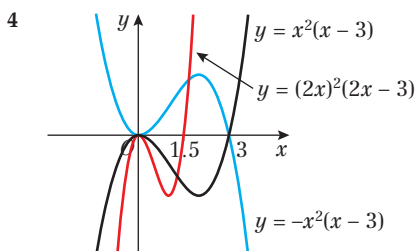
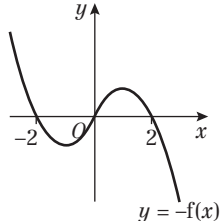
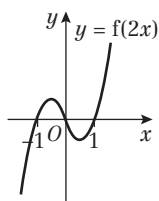
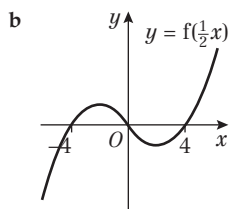
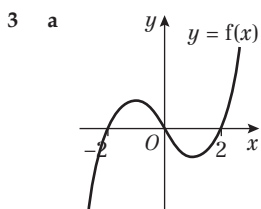
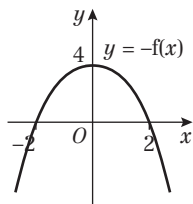
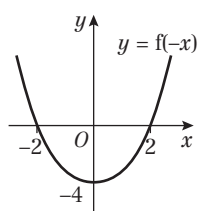
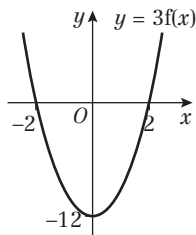
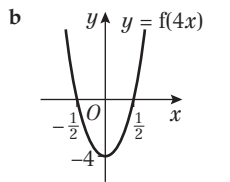
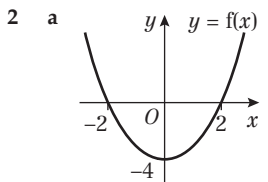
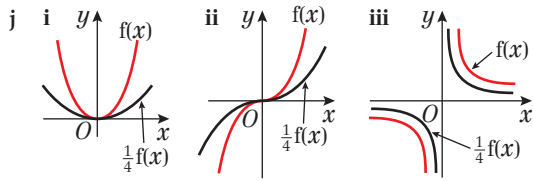
Sfidë

1 $(3, 2)$

2 a $(-7, -12)$ b $f(x-2) + 1$

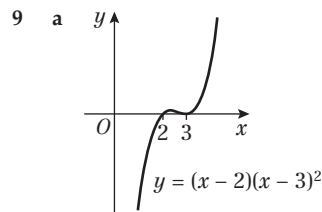
Ushtrime 4F





7 a (1, -3) b (2, -12)

8 (-4, 8)



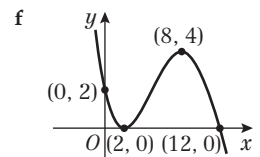
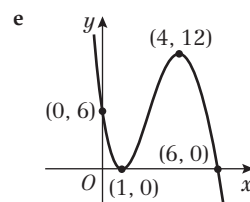
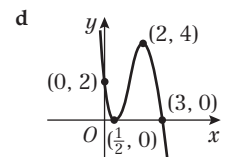
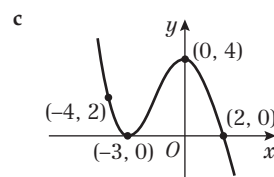
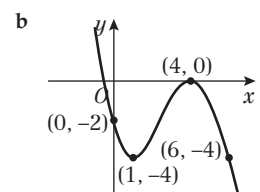
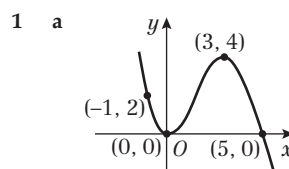
b 2 dhe 3

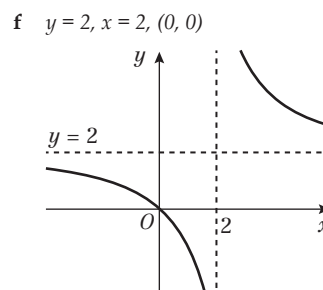
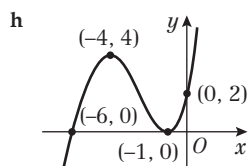
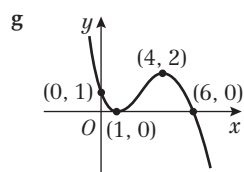
Sfidë

1 (2, -2)

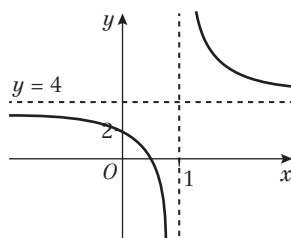
2 $\frac{1}{4}f(\frac{1}{2}x)$

Ushtime 4G

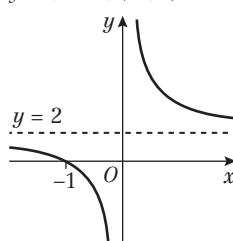




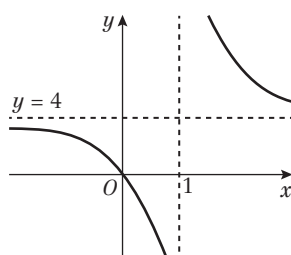
2 a $y = 4, x = 1, (0, 2)$



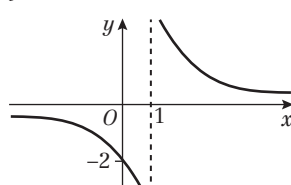
b $y = 2, x = 0, (-1, 0)$



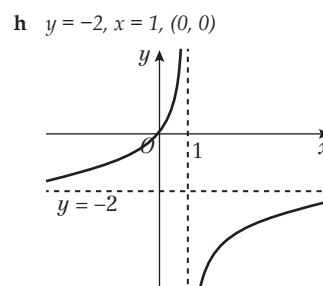
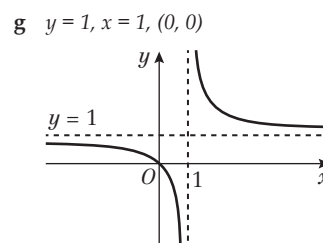
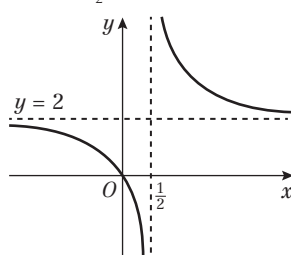
c $y = 4, x = 1, (0, 0)$



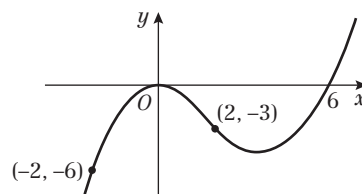
d $y = 0, x = 1, (0, -2)$



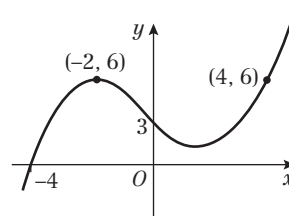
e $y = 2, x = \frac{1}{2}, (0, 0)$



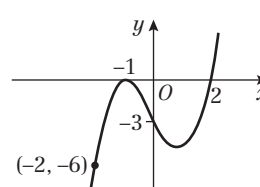
3 a $A(-2, -6), B(0, 0), C(2, -3), D(6, 0)$



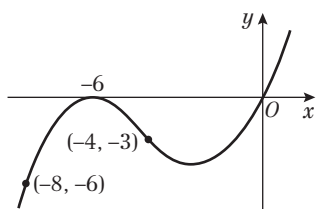
b $A(-4, 0), B(-2, 6), C(0, 3), D(4, 6)$



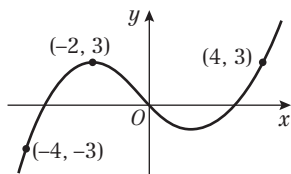
c $A(-2, -6), B(-1, 0), C(0, -3), D(2, 0)$



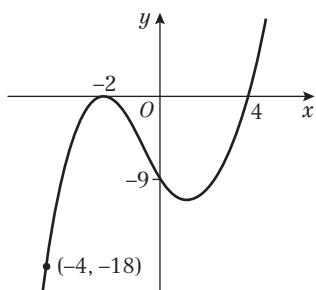
d $A(-8, -6), B(-6, 0), C(-4, -3), D(0, 0)$



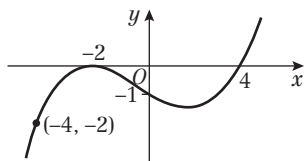
e $A(-4, -3), B(-2, 3), C(0, 0), D(4, 3)$



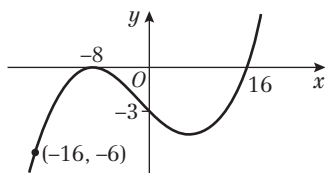
f $A(-4, -18), B(-2, 0), C(0, -9), D(4, 0)$



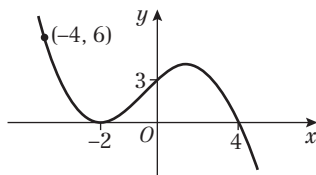
g $A(-4, -2), B(-2, 0), C(0, -1), D(4, 0)$



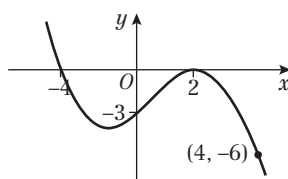
h $A(-16, -6), B(-8, 0), C(0, -3), D(16, 0)$



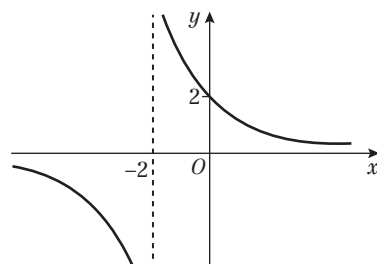
i $A(-4, 6), B(-2, 0), C(0, 3), D(4, 0)$



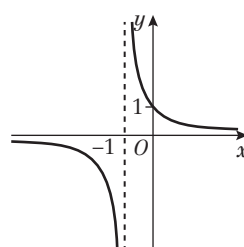
j $A(4, -6), B(2, 0), C(0, -3), D(-4, 0)$



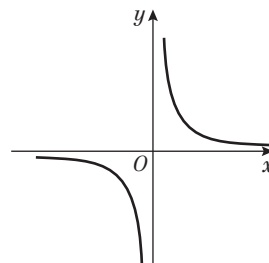
4 a i $x = -2, y = 0, (0, 2)$



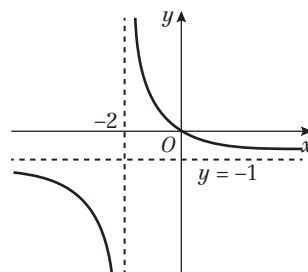
ii $x = -1, y = 0, (0, 1)$



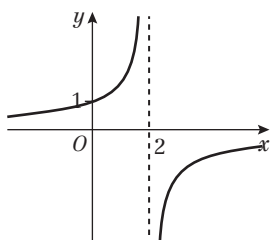
iii $x = 0, y = 0$



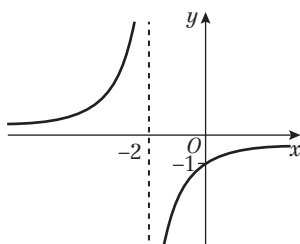
iv $x = -2, y = -1, (0, 0)$



v $x = 2, y = 0, (0, 1)$



vi $x = -2, y = 0, (0, -1)$

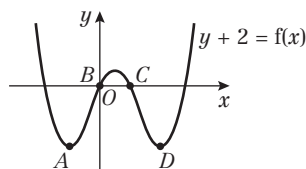


b $f(x) = \frac{2}{x+2}$

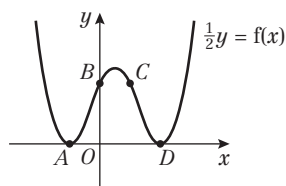
5 a $\frac{1}{2}$

b i (6, 1) ii (2, 3) iii (2, -3.5)

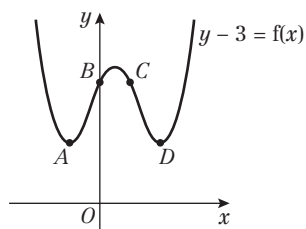
6 a $A(-1, -2) B(0, 0) C(1, 0) D(2, -2)$



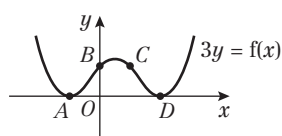
b $A(-1, 0) B(0, 4) C(1, 4) D(2, 0)$



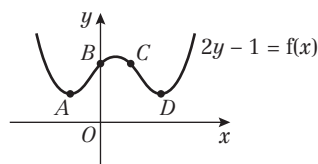
c $A(-1, 3) B(0, 5) C(1, 5) D(2, 3)$



d $A(-1, 0) B(0, \frac{2}{3}) C(1, \frac{2}{3}) D(2, 0)$



e $A(-1, 0.5) B(0, 1.5) C(1, 1.5) D(2, 0.5)$



KAPITULLI 5

Kontrolli i njohurive të mëparshme

- 1 a $(-2, -1)$ b $\frac{9}{69}, \frac{26}{19}$ c $(7, 3)$
 2 a $4\sqrt{5}$ b $10\sqrt{2}$ c $5\sqrt{5}$
 3 a $y = 5 - 2x$ b $y = \frac{2}{5}x - \frac{9}{5}$ c $y = \frac{3}{7}x + \frac{12}{7}$

Ushtrime 5A

- 1 a $\frac{1}{2}$ b $\frac{1}{6}$ c $-\frac{3}{5}$ d 2
 e -1 f $\frac{1}{2}$ g $\frac{1}{2}$ h 8
 i $\frac{2}{3}$ j -4 k $-\frac{1}{3}$ l $-\frac{1}{2}$
 m 1 n $\frac{q^2 - p^2}{q - p} = q + p$

- 2 7
 3 12
 4 $4\frac{1}{3}$
 5 $2\frac{1}{4}$
 6 $\frac{1}{4}$
 7 26
 8 -5
 9 Koeficienti këndor i tangjentes AB = Koeficienti këndor i BC = 0.5; pika B është e përbashkët.
 10 Koeficienti këndor i tangjentes AB = Koeficienti këndor i BC = -0.5; pika B është e përbashkët.

Ushtrime 5B

- 1 a -2 b -1 c 3 d $\frac{1}{3}$
 e $-\frac{2}{3}$ f $\frac{5}{4}$ g $\frac{1}{2}$ h 2
 i $\frac{1}{2}$ j $\frac{1}{2}$ k -2 l $-\frac{3}{2}$
 2 a 4 b -5 c $-\frac{2}{3}$ d 0
 e $\frac{7}{5}$ f 2 g 2 h -2
 i 9 j -3 k $\frac{3}{2}$ l $-\frac{1}{2}$
 3 a $4x - y + 3 = 0$ b $3x - y - 2 = 0$
 c $6x + y - 7 = 0$ d $4x - 5y - 30 = 0$
 e $5x - 3y + 6 = 0$ f $7x - 3y = 0$
 g $14x - 7y - 4 = 0$ h $27x + 9y - 2 = 0$
 i $18x + 3y + 2 = 0$ j $2x + 6y - 3 = 0$
 k $4x - 6y + 5 = 0$ l $6x - 10y + 5 = 0$
 4 (3, 0)
 5 (0, 0)
 6 (0, 5), (-4, 0)
 7 a $\frac{1}{3}$ b $x - 3y + 15 = 0$
 8 a $-\frac{2}{5}$ b $2x + 5y - 10 = 0$
 9 $ax + by + c = 0$
 $by = -ax - c$
 $y = \left(-\frac{a}{b}\right)x - \frac{c}{b}$
 10 $a = 6, c = 10$
 11 $P(3, 0)$

12 a -16 b -27

Sfidë

Koeficienti këndor i tangjentes = $-\frac{a}{b}$; pikëprerja me boshtin $y = a$.
Pra $y = -\frac{a}{b}x + a$

Rishikruaj që të kesh $ax + by - ab = 0$

Ushtrime 5C

- 1 a $y = 2x + 1$ b $y = 3x + 7$ c $y = -x - 3$
d $y = -4x - 11$ e $y = \frac{1}{2}x + 12$ f $y = -\frac{2}{3}x - 5$
g $y = 2x$ h $y = -\frac{1}{2}x + 2b$
- 2 a $y = 4x - 4$ b $y = x + 2$ c $y = 2x + 4$
d $y = 4x - 23$ e $y = x - 4$ f $y = \frac{1}{2}x + 1$
g $y = -4x - 9$ h $y = -8x - 33$ i $y = \frac{6}{5}x$
j $y = \frac{2}{7}x + \frac{5}{14}$
- 3 $5x + y - 37 = 0$
- 4 $y = x + 2$, $y = -\frac{1}{6}x - \frac{1}{3}$, $y = -6x + 23$
- 5 $a = 3$, $c = -27$
- 6 $a = -4$, $b = 8$

Sfidë

- a $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$
- b $y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$
 $\frac{(y - y_1)}{(y_2 - y_1)} = \frac{(x - x_1)}{(x_2 - x_1)}$
- c $y = \frac{3}{7}x + \frac{52}{7}$

Ushtrime 5D

- 1 $y = 3x - 6$
- 3 $2x - 3y + 24 = 0$
- 5 $(-3, 0)$
- 7 $(0, \frac{3}{2})$
- 9 $2x + 3y - 12 = 0$
- 11 $y = \frac{4}{3}x - 4$
- 13 $y = -\frac{4}{5}x + 4$
- 15 $y = -\frac{3}{8}x + \frac{1}{2}$
- 2 $y = 2x + 8$
- 4 $-\frac{1}{5}$
- 6 $(0, 1)$
- 8 $y = \frac{2}{5}x + 3$
- 10 $\frac{8}{5}$
- 12 $6x + 15y - 10 = 0$
- 14 $x - y + 5 = 0$
- 16 $y = 4x + 13$

Ushtrime 5E

- 1 a Paralel b jo paralel c jo paralel
- 2 $r: y = \frac{4}{5}x + 3.2$, $s: y = \frac{4}{5}x - 7$
Koeficientet këndore janë të barabartë, rrjedhimisht drejtëzat janë paralele.
- 3 Koeficienti këndor i $AB = \frac{3}{5}$, Koeficienti këndor i $BC = -\frac{7}{2}$, Koeficienti këndor i $CD = \frac{3}{5}$, Koeficienti këndor i $AD = \frac{10}{3}$.
Katërkëndëshi ka dy brinjë paralele, pra ai është një trapez.
- 4 $y = 5x + 3$
- 5 $2x + 5y + 20 = 0$
- 6 $y = -\frac{2}{3}x + 7$
- 7 $y = \frac{2}{3}x$
- 8 $4x - y + 15 = 0$

Ushtrime 5F

- 1 a Perpendikular b Paralel
c Asnjëra d Perpendikular
e Perpendikular f Paralel
g Paralel h Perpendikular
i Perpendikular j Paralel
k Asnjëra l Perpendikular
- 2 $y = -\frac{1}{6}x + 1$
- 3 $y = \frac{8}{3}x - 8$
- 4 $y = -\frac{1}{3}x$
- 5 $y = -\frac{1}{3}x + \frac{13}{3}$
- 6 $y = -\frac{3}{2}x + \frac{17}{2}$
- 7 $3x + 2y - 5 = 0$
- 8 $7x - 4y + 2 = 0$
- 9 l e ka koeficientin këndor $-\frac{1}{3}$ dhe n e ka koeficientin këndor 3. Koeficientet këndore janë të anasjelltë të njëri-tjetrit dhe me shenjë të kundërt. Atëherë drejtëzat janë perpendikulare.
- 10 $AB: y = -\frac{1}{2}x + 4\frac{1}{2}$, $CD: y = -\frac{1}{2}x - \frac{1}{2}$, $AD: y = 2x + 7$, $BC: y = 2x - 13$. Dy çifte brinjësh paralele dhe drejtëzat me koeficient këndor 2 dhe $-\frac{1}{2}$ janë perpendikulare, pra ABCD është një drejtkëndësh.
- 11 a $A(\frac{7}{9}, 0)$ b $55x - 25y - 77 = 0$
- 12 $-\frac{9}{4}$

Ushtrime 5G

- 1 a 10 b 13 c 5 d $\sqrt{5}$
e $\sqrt{106}$ f $\sqrt{113}$
- 2 Distanca midis A dhe B = $\sqrt{50}$ dhe distanca midis B dhe C = $\sqrt{50}$ pra drejtëzat janë kongruente.
- 3 Distanca midis P dhe Q = $\sqrt{74}$ dhe distanca midis Q dhe R = $\sqrt{73}$ pra drejtëzat nuk janë kongruente.
- 4 $x = -8$ ose $x = 6$
- 5 $y = -2$ ose $y = 16$
- 6 a Të dy drejtëzat e kanë koeficientin këndor 2.
b $y = -\frac{1}{2}x + \frac{23}{2}$ ose $x + 2y - 23 = 0$
c $(\frac{29}{5}, \frac{43}{5})$
d $\frac{7\sqrt{5}}{5}$
- 7 $P(-\frac{3}{5}, \frac{29}{5})$ ose $P(3, -5)$
- 8 a $AB = \sqrt{178}$, $BC = 3$ dhe $AC = \sqrt{205}$. Të gjitha brinjët kanë gjatësi të ndryshme, rrjedhimisht trekëndëshi është brinjëndryshëm.
b $\frac{39}{2}$ ose 19.5
- 9 a $A(2, 11)$
b $B(\frac{41}{4}, 0)$
c $\frac{451}{8}$
- 10 a $(\frac{5}{2}, 0)$ b $(-5, 0)$
c $(-10, -10)$ d $\frac{75}{2}$
- 11 a $y = \frac{1}{2}x - \frac{9}{2}$ b $y = -2x + 8$
c $T(0, 8)$ d $RS = 2\sqrt{5}$ dhe $TR = 5\sqrt{5}$
e 25
- 12 a $x + 4y - 52 = 0$ b $A(0, 13)$
c $B(4, 12)$ d 26



KAPITULLI 6

Kontrolli i njohurive të mëparshme

- 1 a $(x+5)^2+3$ b $(x-3)^2-8$
 c $(x-6)^2-36$ d $(x+\frac{3}{2})^2-\frac{49}{4}$
 2 a $y=\frac{9}{4}x-6$ b $y=-\frac{1}{2}x-\frac{3}{2}$
 c $y=\frac{4}{3}x+\frac{10}{3}$
 3 a $b^2-4ac=-7$ Nuk ka zgjidhje reale
 b $b^2-4ac=89$ Dy zgjidhje reale
 c $b^2-4ac=0$ Një zgjidhje reale
 4 $y=\frac{5}{6}x-\frac{3}{2}$

Ushtrime 6A

- 1 a (5, 5) b (6, 4) c (-1, 4) d (0, 0)
 e (2, 1) f $(-8, \frac{3}{2})$ g (4a, 0) h $(\frac{u}{2}, -v)$
 i $(2a, a-b)$ j $(3\sqrt{2}, 4)$ k $(2\sqrt{2}, \sqrt{2}+3\sqrt{3})$
 2 $a=10, b=1$
 3 $(\frac{3}{2}, 7)$
 4 $(\frac{3a}{5}, \frac{b}{4})$
 5 a $(\frac{3}{2}, 3)$ ose (1.5, 3) b $y=2x, 3=2 \times 1.5$
 6 a $(\frac{1}{8}, \frac{5}{3})$ b $\frac{2}{3}$
 7 Qendra është $(3, -\frac{7}{2})$. $3-2(-\frac{7}{2})-10=0$
 8 (10, 5)
 9 $(-7a, 17a)$
 10 $p=8, q=7$
 11 $a=-2, b=4$

Sfidë

- a $p=9, q=-1$
 b $y=-x+13$
 c AC: $y=-x+8$. Drejtëzat kanë të njëjtin koeficient këndor, pra ato janë paralele.

Ushtrime 6B

- 1 a $y=2x+3$ b $y=-\frac{1}{3}x+\frac{47}{3}$ c $y=\frac{5}{2}x-25$
 d $y=3$ e $y=-\frac{3}{4}x+\frac{37}{8}$ f $x=9$
 2 $y=-x+7$
 3 $2x-y-8=0$
 4 a $y=-\frac{5}{3}x-\frac{13}{3}$ b $y=3x-8$ c $(\frac{11}{4}, -\frac{79}{14})$
 5 $q=-\frac{5}{4}, b=-\frac{189}{8}$

Sfidë

- a $PR: y=-\frac{5}{2}x+\frac{9}{4}$
 $PQ: y=-\frac{1}{4}x+\frac{33}{8}$
 $RQ: y=2x+6$
 b $(-\frac{5}{6}, \frac{13}{3})$

Ushtrime 6C

- 1 a $(x-3)^2+(y-2)^2=16$
 b $(x+4)^2+(y-5)^2=36$
 c $(x-5)^2+(y+6)^2=12$
 d $(x-2a)^2+(y-7a)^2=25a^2$
 e $(x+2\sqrt{2})^2+(y+3\sqrt{2})^2=1$
 2 a $(-5, 4), 9$ b $(7, 1), 4$
 c $(-4, 0), 5$ d $(-4a, -a), 12a$
 e $(3\sqrt{5}, -\sqrt{5}), 3\sqrt{3}$

- 3 a $(4-2)^2+(8-5)^2=4+9=13$
 b $(0+7)^2+(-2-2)^2=49+16=65$
 c $7^2+(-24)^2=49+576=625=25^2$
 d $(6a-2a)^2+(-3a+5a)^2=16a^2+4a^2=20a^2$
 e $(\sqrt{5}-3\sqrt{5})^2+(-\sqrt{5}-\sqrt{5})^2=(-2\sqrt{5})^2+(-2\sqrt{5})^2$
 $=20+20=40=(2\sqrt{10})^2$
 4 $(x-8)^2+(y-1)^2=25$
 5 $(x-\frac{3}{2})^2+(y-4)^2=\frac{65}{4}$
 6 $\sqrt{5}$
 7 a $r=2$
 b Distanca $PQ=PR=RQ=2\sqrt{3}$, tre gjatësi brinjësh të barabarta, rrjedhimisht trekëndëshi është barabrinjës.
 8 a $(x-2)^2+y^2=15$
 b Qendra (2, 0) dhe rrezja $=\sqrt{15}$
 9 a $(x-5)^2+(y+2)^2=49$
 b Qendra (5, -2) dhe rrezja = 7
 10 a Qendra (1, -4), dhe rrezja 5
 b Qendra (-6, 2), dhe rrezja 7
 c Qendra (11, 3), dhe rrezja $3\sqrt{10}$
 d 10 Qendra (-2.5, 1.5), dhe rrezja $\frac{5\sqrt{2}}{2}$
 e Qendra (2, -2), dhe rrezja $\sqrt{6.5}$
 11 a Qendra (-6, -1)
 b $k > -37$
 12 $Q(-13, 28)$
 13 $k=-2$ dhe $k=8$

Sfidë

- 1 $k = 3, (x - 3)^2 + (y - 2)^2 = 50$
 $k = 5, (x - 5)^2 + (y - 2)^2 = 50$
- 2 $(x + f)^2 - f^2 + (y + g)^2 - g^2 + c = 0$
 So $(x + f)^2 + (y + g)^2 = f^2 + g^2 - c$
 Rrethi me qendër $(-f, -g)$ dhe rreze $\sqrt{f^2 + g^2 - c}$.

Ushtrime 6D

- 1 $(7, 0), (-5, 0)$
 2 $(0, 2), (0, -8)$
 3 $(6, 10), (-2, 2)$
 4 $(4, -9), (-7, 2)$
 5 $2x^2 - 24x + 79 = 0$ nuk ka zgjidhje reale, rrjedhimisht drejtëzat nuk presin rrethin.
 6 a $b^2 - 4ac = 64 - 4 \times 1 \times 16 = 0$. Rrjedhimisht ka vetëm një pikë prerje.
 b $(4, 7)$
 7 a $(0, -2), (4, 6)$ b pika e mesit e AB është $(2, 2)$
 8 a 13 b $p = 1$ ose 5
 9 a $A(5, 0)$ dhe $B(-3, -8)$ (ose anasjelltas)
 b $y = -x - 3$
 c $(4, -7)$ është zgjidhje e $y = -x - 3$.
 d 20
 10 a Zëvendëso $y = kx$ dhe gjen
 $(k^2 + 1)x^2 - (12k + 10)x + 57 = 0$
 $b^2 - 4ac > 0, -84k^2 + 240k - 128 > 0,$
 $21k^2 - 60k + 32 < 0$
 b $0.71 < k < 2.15$
 Përgjigja e saktë është $\frac{10}{7} - \frac{2\sqrt{57}}{21} < k < \frac{10}{7} + \frac{2\sqrt{57}}{21}$
 11 $k < \frac{8}{17}$
 12 $k = -20 \pm 2\sqrt{105}$

Ushtrime 6E

- 1 a $3\sqrt{10}$
 b Koeficienti këndor i rrezes = 3, koeficienti këndor i drejtëzës = $-\frac{1}{3}$, koeficientet këndore janë të anasjelltë të njëri-tjetrit dhe me shenjë të kundërt dhe rrjedhimisht perpendikulare.
 2 a $(x - 4)^2 + (y - 6)^2 = 73$ b $3x + 8y + 13 = 0$
 3 a $y = -2x - 1$
 b Qendra e rrethit $(1, -3)$ kënaq $y = -2x - 1$.
 4 a $y = \frac{1}{2}x - 3$
 b Qendra e rrethit $(2, -2)$ kënaq $y = \frac{1}{2}x - 3$
 5 a $(-7, -6)$ kënaq $x^2 + 18x + y^2 - 2y + 29 = 0$
 b $y = \frac{2}{7}x - 4$ c $R(0, -4)$ d $\frac{53}{2}$
 6 a $(0, -17), (17, 0)$
 b 144.5
 7 $y = 2x + 27$ dhe $y = 2x - 13$
 8 a $p = 4, p = -6$
 b $(3, 4)$ dhe $(3, -6)$
 9 a $(x - 11)^2 + (y + 5)^2 = 100$
 b $y = \frac{3}{4}x - \frac{3}{4}$
 c $A(8 - 4\sqrt{3}, -1 - 3\sqrt{3})$ dhe $B(8 + 4\sqrt{3}, -1 + 3\sqrt{3})$
 d $10\sqrt{3}$
 10 a $y = 4x - 22$
 b $a = 5$
 c $(x - 5)^2 + (y + 2)^2 = 34$
 d $A(5 + \sqrt{2}, -2 + 4\sqrt{2})$ dhe $B(5 - \sqrt{2}, -2 - 4\sqrt{2})$

- 11 a $P(-2, 5)$ dhe $Q(4, 7)$
 b $y = 2x + 9$ dhe $y = -\frac{1}{2}x + 9$
 c $y = -3x + 9$
 d $(0, 9)$

Sfidë

- 1 $y = \frac{1}{2}x - 2$
 2 a $\angle CPR = \angle CQR = 90^\circ$ (Këndi midis tangjentes dhe rrezes)
 $CP = CQ = \sqrt{10}$ (Rrezja e rrethit)
 $CR = \sqrt{(6 - 2)^2 + (-1 - 1)^2} = \sqrt{20}$
 (Pra nga teorema e Pitagorës,
 $PR = QR = \sqrt{20} - 10 = \sqrt{10}$
 4 brinjë të barabarta dhe dy kënde të kundërt të drejtë
 pra $CPRQ$ është një katror.
 b $y = \frac{1}{3}x - 3$ dhe $y = -3x + 17$

Ushtrime 6F

- 1 a $WV^2 = WU^2 + UV^2$
 b $(2, 3)$
 c $(x - 2)^2 + (y - 3)^2 = 41$
 2 a $AC^2 = AB^2 + BC^2$
 b $(x - 5)^2 + (y - 2)^2 = 25$
 c 15
 3 a i $y = \frac{3}{2}x + \frac{21}{2}$ ii $y = -\frac{2}{3}x + 4$
 b $(-3, 6)$
 c $(x + 3)^2 + (y - 6)^2 = 169$
 4 a i $y = \frac{1}{3}x + \frac{10}{3}$ ii $x = -1$
 b $(x + 1)^2 + (y - 3)^2 = 125$
 5 $(x - 3)^2 + (y + 4)^2 = 50$
 6 a $AB^2 + BC^2 = AC^2$
 $AB^2 = 400, BC^2 = 100, AC^2 = 500$
 b $(x + 2)^2 + (y - 5)^2 = 125$
 c $D(8, 0)$ kënaq ekuacionin e rrethit.
 7 a $AB = BC = CD = DA = \sqrt{50}$
 b 50
 c $(3, 6)$
 8 a $DE^2 = b^2 + 6b + 13$
 $EF^2 = b^2 + 10b + 169$
 $DF^2 = 200$
 So $b^2 + 6b + 13 + b^2 + 10b + 169 = 200$
 $(b + 9)(b - 1) = 0$; as $b > 0, b = 1$
 b $(x + 5)^2 + (y + 4)^2 = 50$
 9 a Qendra $(-1, 12)$ dhe rrezja = 13
 b Me anë të formulës së distancës gjej $AB = 26$. Kjo është dy herë sa rrezja, pra AB është një diametër. Ka edhe metoda të tjera të mundshme.
 c $C(-6, 0)$

KAPITULLI 7

Kontrolli i njohurive të mëparshme

- 1 a $15x^7$ b $\frac{x}{3y}$
 2 a $(x - 6)(x + 4)$ b $(3x - 5)(x - 4)$
 3 a 8567 b 1652
 4 a $y = 1 - 3x$ b $y = \frac{1}{2}x - 7$
 5 a $(x - 1)^2 - 21$ b $2(x + 1)^2 + 13$

Ushtrime 7A

- 1 a $4x^3 + 5x - 7$ b $2x^4 + 9x^2 + x$
 c $-x^3 + 4x + \frac{6}{x}$ d $7x^4 - x^2 - \frac{4}{x}$
 e $4x^3 - 2x^2 + 3$ f $3x - 4x^2 - 1$



g $\frac{7x^2}{5} - \frac{x^3}{5} - \frac{2}{5x}$

i $\frac{x^7}{2} - \frac{9x^3}{2} + 2x^2 - \frac{3}{x}$

2 a $x + 3$

d $x + 7$

g $\frac{x-4}{x-3}$

j $\frac{2x+3}{x-5}$

m $\frac{2x+1}{x-2}$

b $x + 4$

e $x + 5$

h $\frac{x+2}{x+4}$

k $\frac{2x-3}{x+1}$

n $\frac{x+4}{3x+1}$

h $2x - 3x^3 + 1$

j $3x^8 + 2x^5 - \frac{4x^3}{3} + \frac{2}{3x}$

c $x + 3$

f $x + 4$

i $\frac{x+4}{x-6}$

l $\frac{x-2}{x+2}$

o $\frac{2x+1}{2x-3}$

3 $a = 1, b = 4, c = -2$

Ushtrime 7B

1 a $(x+1)(x^2+5x+3)$

c $(x+2)(x^2-3x+7)$

e $(x-5)(x^2-3x-2)$

2 a $(x+4)(6x^2+3x+2)$

c $(x+3)(2x^2-2x-3)$

e $(x+6)(-5x^2+3x+5)$

3 a x^3+3x^2-4x+1

c $-3x^3+3x^2-4x-7$

4 a x^3+2x^2-5x+4

c $2x^3+5x+2$

e $2x^4-2x^3+3x^2+4x-7$

g $5x^3+12x^2-6x-2$

5 a x^2-2x+5

c $-3x^2-12x+2$

6 a $x^2+4x+12$

c $-3x^2+5x+10$

7 Pjesëto $x^3+2x^2-5x-10$ me $(x+2)$ dhe gjej (x^2-5) . Pra $x^3+2x^2-5x-10 = (x+2)(x^2-5)$.

8 a -8

b -7

c -12

9 $f(1) = 3 - 2 + 4 = 5$

10 $f(-1) = 3 + 8 + 10 + 3 - 25 = -1$

11 $(x+4)(5x^2-20x+7)$

12 $3x^2+6x+4$

13 x^2+x+1

14 x^3-2x^2+4x-8

15 14

16 a -200

b $(x+2)(x-7)(3x+1)$

17 a i 30

ii 0

b $x = -3, x = -4, x = 1$

18 a $a = 1, b = 2, c = -3$

b $f(x) = (2x-1)(x+3)(x-1)$

c $x = 0.5, x = -3, x = 1$

19 a $a = 3, b = 2, c = 1$

b Shprehja kuadratike nuk ka zgjidhje reale, pra vetëm $\frac{1}{4}$ është një zgjidhje.

Ushtrime 7C

1 a $f(1) = 0$

b $f(-3) = 0$

c $f(4) = 0$

2 $(x-1)(x+3)(x+4)$

3 $(x+1)(x+7)(x-5)$

4 $(x-5)(x-4)(x+2)$

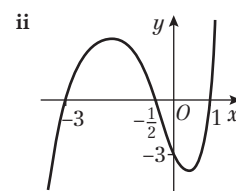
5 $(x-2)(2x-1)(x+4)$

6 a $(x+1)(x-5)(x-6)$

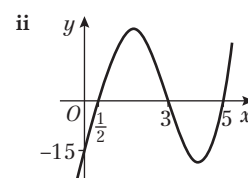
b $(x-2)(x+1)(x+2)$

c $(x-5)(x+3)(x-2)$

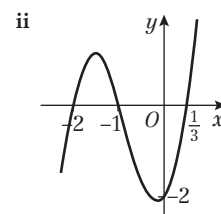
7 a i $(x-1)(x+3)(2x+1)$



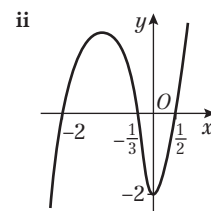
b i $(x-3)(x-5)(2x-1)$



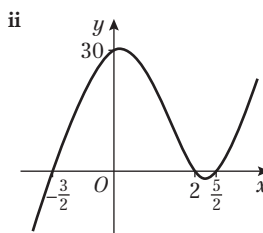
c i $(x+1)(x+2)(3x-1)$



d i $(x+2)(2x-1)(3x+1)$



e i $(x-2)(2x-5)(2x+3)$



8 2

9 -16

10 $p = 3, q = 7$

11 $c = 2, d = 3$

12 $g = 3, h = -7$

13 a $f(4) = 0$

b $f(x) = (x-4)(3x^2+6)$

For $3x^2+6=0$, $b^2-4ac = -72$ pra there are no real roots.Therefore, 4 është the only real root of $f(x) = 0$.

14 a $f(-2) = 0$

b $(x+2)(2x+1)(2x-3)$

c $x = -2, x = -\frac{1}{2}$ dhe $x = \frac{1}{2}$

15 a $f(2) = 0$

b $x = 0, x = 2, x = -\frac{1}{3}$ dhe $x = \frac{1}{3}$

Sfidë

a $f(1) = 2 - 5 - 42 - 9 + 54 = 0$

$f(-3) = 162 + 135 - 378 + 27 + 54 = 0$

b $2x^4 - 5x^3 - 42x^2 - 9x + 54$

$= (x-1)(x+3)(x-6)(2x+3)$

$x = 1, x = -3, x = 6, x = -1.5$

Ushtrime 7D

- 1 $n^2 - n = n(n-1)$
Në qoftë se n është çift, atëherë, $n-1$ është tek dhe çift \times tek = çift.
Në qoftë se n është tek atëherë, $n-1$ është çift dhe tek \times çift = çift
- 2 $\frac{x}{(1+\sqrt{2})} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})} = \frac{x(1-\sqrt{2})}{(1-2)} = \frac{x-x\sqrt{2}}{-1} = x\sqrt{2} - x$
- 3 $(x+\sqrt{y})(x-\sqrt{y}) = x^2 - x\sqrt{y} + x\sqrt{y} - y = x^2 - y$
- 4 $(2x-1)(x+6)(x-5) = (2x-1)(x^2+x-30) = 2x^3+x^2-61x+30$
- 5 Ana e majtë = $x^2 + bx$, duke përdorur plotësimin e katrorit,
 $(x+\frac{b}{2})^2 - \frac{b^2}{4}$
- 6 $x^2 + 2bx + c = 0$, duke përdorur plotësimin e katrorit,
 $(x+b)^2 + c - b^2 = 0$
 $(x+b)^2 = b^2 - c$
 $x+b = \pm \sqrt{b^2 - c}$
 $x = -b \pm \sqrt{b^2 - c}$
- 7 $(x - \frac{2}{x})^3 = (x - \frac{2}{x})(x^2 - 4 + \frac{4}{x^2}) = x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}$
- 8 $(x^3 - \frac{1}{x})(x^3 + x^{\frac{1}{2}}) = x^6 + x^{\frac{3}{2}} - x^{\frac{1}{2}} - x^{-\frac{7}{2}} = x^6 - x^{-\frac{7}{2}}$
 $= x^6(x^4 - \frac{1}{x^4})$
- 9 $3n^2 - 4n + 10 = 3[n^2 - \frac{4}{3}n + \frac{10}{3}] = 3[(n - \frac{2}{3})^2 + \frac{10}{3} - \frac{4}{9}]$
 $= 3(n - \frac{2}{3})^2 + \frac{26}{3}$
Vlera minimale është $\frac{26}{3}$ pra $3n^2 - 4n + 10$ është gjithnjë pozitive.
- 10 $-n^2 - 2n - 3 = -[n^2 + 2n + 3] = -[(n+1)^2 + 3 - 1]$
 $= -(n+1)^2 - 2$
Vlera maksimale është -2 pra $-n^2 - 2n - 3$ është gjithnjë negative.
- 11 $x^2 + 8x + 20 = (x+4)^2 + 4$
Vlera minimale është 4 pra $x^2 + 8x + 20$ është përherë më e madhe ose e barabartë me 4.
- 12 $kx^2 + 5kx + 3 = 0$, $b^2 - 4ac < 0$, $25k^2 - 12k < 0$,
 $k(25k - 12) < 0$, $0 < k < \frac{12}{25}$
Kur $k = 0$ nuk ka rrënjë reale, pra $0 \leq k < \frac{12}{25}$
- 13 $px^2 - 5x - 6 = 0$, $b^2 - 4ac > 0$, $25 + 24p > 0$, $p > -\frac{25}{24}$
- 14 Koeficienti këndor $AB = -\frac{1}{2}$, koeficienti këndor $BC = 2$,
Koeficienti këndor $AB \times$ koeficienti këndor $BC = -\frac{1}{2} \times 2 = -1$,
pra AB dhe BC janë paralele.
- 15 Koeficienti këndor $AB = 3$, koeficienti këndor $BC = \frac{1}{4}$, koeficienti këndor $CD = 3$, koeficienti këndor $AD = \frac{1}{4}$
Koeficienti këndor $AB =$ koeficienti këndor CD pra AB dhe CD janë paralele.
Koeficienti këndor $BC =$ koeficienti këndor AD pra BC dhe AD janë paralele.
- 16 Koeficienti këndor $AB = \frac{1}{3}$, koeficienti këndor $BC = 3$, koeficienti këndor $CD = \frac{1}{3}$, koeficienti këndor $AD = 3$
Koeficienti këndor $AB =$ koeficienti këndor CD pra AB dhe CD janë paralele.
Koeficienti këndor $BC =$ koeficienti këndor AD pra BC dhe AD janë paralele.
Gjatësi $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$ dhe $AD = \sqrt{10}$,
pra të katër brinjët janë të barabarta..

- 17 Koeficienti këndor $AB = -3$, koeficienti këndor $BC = \frac{1}{3}$,
Koeficienti këndor $AB \times$ koeficienti këndor $BC = -3 \times \frac{1}{3} = -1$,
pra AB dhe BC janë paralele.
Gjatësia $AB = \sqrt{40}$, $BC = \sqrt{40}$, $AB = BC$
- 18 $(x-1)^2 + y^2 = k$, $y = ax$, $(x-1)^2 + a^2x^2 = k$,
 $x^2(1+a^2) - 2x + 1 - k = 0$
 $b^2 - 4ac > 0$, $k > \frac{a^2}{1+a^2}$
- 19 $x = 2$. Ka vetëm një zgjidhje, pra drejtëza
 $4y - 3x + 26 = 0$ vetëm sa e prek rrethin në një pikë, pra është tangent ndaj rrethit.
- 20 Syprina e katrorit = $(a+b)^2 = a^2 + 2ab + b^2$
Syprina e ngjyrosur = $4ab$
Syprina e katrorit të vogël: $a^2 + 2ab + b^2 - 2ab = a^2 + b^2 = c^2$

Sfidë

- 1 Ekuacioni i rrethit është $(x-3)^2 + (y-5)^2 = 25$ dhe të katër pikat kënaqin këtë ekuacion.
- 2 $2k+1 = 1 \times (2k+1) = ((k+1)-k)((k+1)+k) = (k+1)^2 - k^2$

Ushtrime 7E

- 1 3, 4, 5, 6, 7 dhe 8 nuk plotpjesëtohen me 10.
- 2 3, 5, 7, 11, 13, 17, 19, 23 janë numra të thjeshtë. 9, 15, 21, 25 janë prodhim i dy numrave të thjeshtë.
- 3 $1^2 + 2^2 = 5$, $2^2 + 3^2 = 13$, $3^2 + 4^2 = 25$, $4^2 + 5^2 = 41$, $5^2 + 6^2 = 61$, $6^2 + 7^2 = 85$, $7^2 + 8^2 = 113$
- 4 $(3n)^3 = 27n^3 = 9n(3n^2)$ që është shumëfish i 9
 $(3n+1)^3 = 27n^3 + 27n^2 + 9n + 1 = 9n(3n^2 + 3n + 1) + 1$ i cili është një më shumë se një shumëfish i 9
 $(3n+2)^3 = 27n^3 + 54n^2 + 36n + 8 = 9n(3n^2 + 6n + 4) + 8$ i cili është një më shumë se një shumëfish i 9
- 5 a Për shembull, kur $n = 2$, $2^4 - 2 = 14$, 14 nuk plotpjesëtohet me 4.
b Çdo numër katror.
c Për shembull, kur $n = \frac{1}{2}$
d Për shembull, kur $n = 1$
- 6 a Duke supozuar se x dhe y janë pozitive.
b p.sh. $x = 0$, $y = 0$
- 7 $(x+5)^2 \geq 0$ për të gjitha vlerat x , dhe
 $(x+5)^2 + 2x + 11 = (x+6)^2$, pra $(x+6)^2 \geq 2x + 11$
- 8 Në qoftë se $a^2 + 1 \geq 2a$ (a është pozitive, pra duke shumëzuar të dy anët me $a^2 - 2a + 1 \geq 0$, nuk ndryshon shenja e inekuacionit), atëherë dhe $(a-1)^2 \geq 0$, dhe ne e dimë se kjo nuk është e vërtetë.
- 9 a $(p+q)^2 = p^2 + 2pq + q^2 = (p-q)^2 + 4pq$
 $(p-q)^2 \geq 0$ sepse është një katror, pra $(p+q)^2 \geq 4pq$
 $p > 0$, $q > 0 \Rightarrow p+q > 0 \Rightarrow p+q \geq \sqrt{4pq}$
b p.sh. $p = q = -1$: $p+q = -2$, $\sqrt{4pq} = 2$
- 10 a Fillo duke supozuar se inekuacioni është i vërtetë: pra negative \geq pozitiv
b p.sh. $x = y = -1$: $x+y = -2$, $\sqrt{x^2+y^2} = \sqrt{2}$
c $(x+y)^2 = x^2 + 2xy + y^2 > x^2 + y^2$ sepse $x > 0$,
 $y > 0 \Rightarrow 2xy > 0$
Me qenë se $x+y > 0$, merr rrënjët katrore: $x+y \geq \sqrt{x^2+y^2}$

Ushtrime 8A

- 1 a Rreshti i 4 b rreshti i 16
c rreshti i $(n+1)$ d rreshti i $(n+5)$
- 2 a $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
b $p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$
c $a^3 - 3a^2b + 3ab^2 - b^3$



- d $x^3 + 12x^2 + 48x + 64$
 e $16x^4 - 96x^3 + 216x^2 - 216x + 81$
 f $a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32$
 g $81x^4 - 432x^3 + 864x^2 - 768x + 256$
 h $16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$
 3 a 16 b -10 c 8 d 1280
 e 160 f -2 g 40 h -96
 4 $1 + 9x + 30x^2 + 44x^3 + 24x^4$
 5 $8 + 12y + 6y^2 + y^3$, $8 + 12x - 6x^2 - 11x^3 + 3x^4 + 3x^5 - x^6$
 6 ± 3
 7 $\frac{5}{2} - 1$
 8 $12p$
 9 $500 + 25X + \frac{X^2}{2}$

Sfidë

 $\frac{3}{4}$

Ushtrime 8B

- 1 a 24 b 362 880 c 720 d 210
 2 a 6 b 15 c 20 d 5
 e 45 f 126
 3 a 5005 b 120 c 184 756 d 1140
 e 2002 f 8568
 4 $a = {}^4C_1$, $b = {}^5C_2$, $c = {}^6C_2$, $d = {}^6C_3$
 5 330
 6 a 120, 210 b 960
 7 a 286, 715 b 57 915
 8 0.1762 me 4 shifra dhjetore. Megjithëse duket probabilitet i vogël, shanset janë më të mëdha që monedha të bjerë 10 herë kokë se sa çdo numër tjetër herësh rëniesh kokë.

- 9 a ${}^nC_1 = \frac{n!}{1!(n-1)!}$

$$= \frac{1 \times 2 \times \dots \times (n-2) \times (n-1) \times n}{1 \times 1 \times 2 \times \dots \times (n-3) \times (n-2) \times (n-1)} = n$$

 b ${}^nC_2 = \frac{n!}{2!(n-2)!}$

$$= \frac{1 \times 2 \times \dots \times (n-2) \times (n-1) \times n}{1 \times 2 \times 1 \times 2 \times \dots \times (n-3) \times (n-2)} = \frac{n(n-1)}{2}$$

 10 $a = 37$
 11 $p = 17$

Sfidë

- a ${}^{10}C_3 = \frac{10!}{3!7!} = 120$ dhe ${}^{10}C_7 = \frac{10!}{7!3!} = 120$
 b ${}^{14}C_5 = \frac{14!}{5!9!} = 2002$ dhe ${}^{14}C_9 = \frac{14!}{9!5!} = 2002$
 c Të dyja përgjigjet për pikën a janë të njëjta dhe të dyja përgjigjet për pikën b janë të njëjta.
 d ${}^nC_r = \frac{n!}{r!(n-r)!}$ dhe ${}^nC_{n-r} = \frac{n!}{(n-r)!r!}$ rrjedhimisht ${}^nC_r = {}^nC_{n-r}$

Ushtrime 8C

- 1 a $1 + 4x + 6x^2 + 4x^3 + x^4$
 b $81 + 108x + 54x^2 + 12x^3 + x^4$
 c $256 - 256x + 96x^2 - 16x^3 + x^4$
 d $x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$
 e $1 + 8x + 24x^2 + 32x^3 + 16x^4$
 f $1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$
 2 a $1 + 10x + 45x^2 + 120x^3$
 b $1 - 10x + 40x^2 - 80x^3$
 c $1 + 18x + 135x^2 + 540x^3$

- d $256 - 1024x + 1792x^2 - 1792x^3$
 e $1024 - 2560x + 2880x^2 - 1920x^3$
 f $2187 - 5103x + 5103x^2 - 2835x^3$
 3 a $64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3$
 b $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3$
 c $p^8 - 8p^7q + 28p^6q^2 - 56p^5q^3$
 d $729x^6 - 1458x^5y + 1215x^4y^2 - 540x^3y^3$
 e $x^8 + 16x^7y + 112x^6y^2 + 448x^5y^3$
 f $512x^9 - 6912x^8y + 41472x^7y^2 - 145152x^6y^3$
 4 a $1 + 8x + 28x^2 + 56x^3$
 b $1 - 12x + 60x^2 - 160x^3$
 c $1 + 5x + \frac{45}{4}x^2 + 15x^3$
 d $1 - 15x + 90x^2 - 270x^3$
 e $128 + 448x + 672x^2 + 560x^3$
 f $27 - 54x + 36x^2 - 8x^3$
 g $64 - 576x + 2160x^2 - 4320x^3$
 h $256 + 256x + 96x^2 + 16x^3$
 i $128 + 2240x + 16800x^2 + 70000x^3$
 5 $64 - 192x + 240x^2$
 6 $243 - 810x + 1080x^2$
 7 $x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}$

Sfidë

- a $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
 $(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$
 $(a+b)^4 - (a-b)^4 = 8a^3b + 8ab^3 = 8ab(a^2 + b^2)$
 b $82896 = 2^4 \times 3 \times 11 \times 157$

Ushtrime 8D

- 1 a 90 b 80 c -20
 d 1080 e 120 f -4320
 g 1140 h -241 920 i -2.5
 j 354.375 k -224 l 3.90625
 2 $a = \pm \frac{1}{2}$
 3 $b = -2$
 4 $\frac{5 \pm \sqrt{105}}{8}$
 5 a $p = 5$ b -10 c -80
 6 a $5^{30} + 5^{29} \times 30px + 5^{28} \times 435p^2x^2$
 b $p = 10$
 7 a $1 + 10qx + 45q^2x^2 + 120q^3x^3$
 b $q = \pm 3$
 8 a $1 + 11px + 55p^2x^2$
 b $p = 7$, $q = 2695$
 9 a $1 + 15px + 105p^2x^2$
 b $p = -\frac{5}{7}$, $q = 10\frac{5}{7}$
 10 $\frac{q}{p} = 2.1$

Sfidë

- a 314 928 b 43 750

KAPITULLI 9

Kontrolli i njohurive të mëparshme

- 1 a 3.10 cm b 9.05 cm
 2 a 25.8° b 77.2°
 3 a grafiku i $x^2 + 3x$ b grafiku i $(x+2)^2 + 3(x+2)$
 c grafiku i $x^2 + 3x - 3$ b grafiku i $(0.5x)^2 + 3(0.5x)$

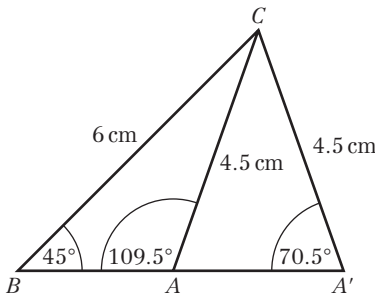
Ushtrime 9A

- 1 a 3.19 cm b 1.73 cm ($\sqrt{3}$ cm) c 9.85 cm
d 4.31 cm e 6.84 cm f 9.80 cm
- 2 a 108(.2)° b 90° c 60°
d 52.6° e 137° f 72.2°
- 3 192 km
- 4 11.2 km
- 5 128.5° ose 031.5° (Këndi BAC = 48.5°)
- 6 302 jard (301.5...)
- 7 Me anë të rregullës së kosinusi $\frac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4} = \frac{1}{8}$
- 8 Me anë të rregullës së kosinusi $\frac{2^2 + 3^2 - 4^2}{2 \times 2 \times 3} = \frac{1}{4}$
- 9 $\angle ACB = 22.3^\circ$
- 10 $\angle ABC = 108(.4)^\circ$
- 11 104(.48)°
- 12 $x = 4.4$ cm
- 13 $x = 42$ cm
- 14 a $y^2 = (5-x)^2 + (4+x)^2 - 2(5-x)(4+x) \cos 120^\circ$
 $= 25 - 10x + x^2 + 16 + 8x + x^2 - 2(20 + x - x^2) \left(-\frac{1}{2}\right)$
 $= x^2 - x + 61$
b Minimumi $AC^2 = 60.75$; ai arrihet kur $x = \frac{1}{2}$
- 15 a $\cos \angle ABC = \frac{x^2 + 5^2 - (10-x)^2}{2x \times 5}$
 $= \frac{20x - 75}{10x} = \frac{4x - 15}{2x}$
b 3.5
- 16 65.3°
- 17 a 28.7 km b 056.6°

Ushtrime 9B

- 1 a 15.2 cm b 9.57 cm c 8.97 cm d 4.61 cm
- 2 a $x = 84^\circ$, $y = 6.32$
b $x = 13.5$, $y = 16.6$
c $x = 85^\circ$, $y = 13.9$
d $x = 80^\circ$, $y = 6.22$ (trekëndësh dybrinjënjëshëm)
e $x = 6.27$, $y = 7.16$
f $x = 4.49$, $y = 7.49$ (trekëndësh kënddrejtë)
- 3 a 36.4° b 35.8° c 40.5° d 130°
- 4 a 48.1° b 45.6° c 14.8° d 48.7°
e 86.5° f 77.4°
- 5 a 1.41 cm ($\sqrt{2}$ cm) b 1.93 cm
- 6 $\angle PQR = 50.6^\circ$, $\angle QPR = 54.4^\circ$
- 7 a $x = 43.2^\circ$, $y = 5.02$ cm b $x = 101^\circ$, $y = 15.0$ cm
c $x = 6.58$ cm, $y = 32.1^\circ$ d $x = 54.6^\circ$, $y = 10.3$ cm
e $x = 21.8^\circ$, $y = 3.01$ f $x = 45.9^\circ$, $y = 3.87^\circ$
- 8 a 6.52 km b 3.80 km
- 9 a 7.31 cm b 1.97 cm
- 10 a 66.3° b 148 m
- 11 Nga rregulla e sinusit, $x = \frac{4\sqrt{2}}{2 + \sqrt{2}}$; duke racionalizuar
 $x = \frac{4\sqrt{2}(2 - \sqrt{2})}{2} = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$.
- 12 a 36.5 m
b Këndet maten nga niveli i tokës.

Ushtrime 9C

- 1 a 70.5°, 109° (109.5°)
b 
- 2 a $x = 74.6^\circ$, $y = 65.4^\circ$
 $x = 105^\circ$, $y = 34.6^\circ$
b $x = 59.8^\circ$, $y = 48.4$ cm
 $x = 120^\circ$, $y = 27.3$ cm
c $x = 56.8^\circ$, $y = 4.37$ cm
 $x = 23.2^\circ$, $y = 2.06$ cm
- 3 a 5 cm ($\angle ACB = 90^\circ$) b 24.6°
c 45.6°, 134(.4)°
- 4 2.97 cm
- 5 Në një trekëndësh $\angle ABC = 101^\circ$ (100.9°); në tjetrin $\angle BAC = 131^\circ$ (130.9°)
- 6 a 62.0° b Lëvizja është simetrike

Ushtrime 9D

- 1 a 23.7 cm² b 4.31 cm² c 20.2 cm²
- 2 a $x = 41.8^\circ$ ose 138(.2)°
b $x = 26.7^\circ$ ose 153(.3)°
c $x = 60^\circ$ ose 120°
- 3 275(.3) m (brinja e tretë = 135.3 m)
- 4 3.58
- 5 a $Syprina = \frac{1}{2}(x+2)(5-x) \sin 30^\circ$
 $= \frac{1}{2}(10 + 3x - x^2) \times \frac{1}{2}$
 $= \frac{1}{4}(10 + 3x - x^2)$
b Maksimumi $A = 3\frac{1}{16}$ kur $x = 1\frac{1}{2}$
- 6 a $\frac{1}{2}x(5+x) \sin 150^\circ = \frac{15}{4}$
 $\frac{1}{2}(5x + x^2) \times \frac{1}{2} = \frac{15}{4}$
 $5x + x^2 = 15$
 $x^2 + 5x - 15 = 0$
b 2.11

Ushtrime 9E

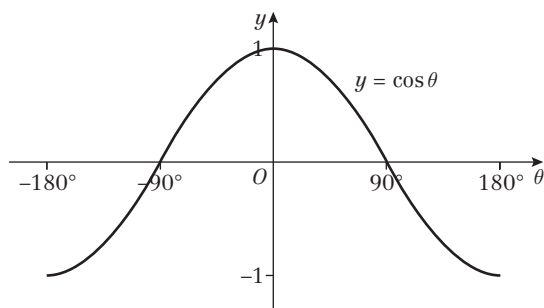
- 1 a $x = 37.7^\circ$, $y = 86.3^\circ$, $z = 6.86$
b $x = 48^\circ$, $y = 19.5$, $z = 14.6$
c $x = 30^\circ$, $y = 11.5$, $z = 11.5$
d $x = 21.0^\circ$, $y = 29.0^\circ$, $z = 8.09$
e $x = 93.8^\circ$, $y = 56.3^\circ$, $z = 29.9^\circ$
f $x = 97.2^\circ$, $y = 41.4^\circ$, $z = 41.4^\circ$
g $x = 45.3^\circ$, $y = 94.7^\circ$, $z = 14.7$
ose $x = 135^\circ$, $y = 5.27^\circ$, $z = 1.36$
h $x = 7.07$, $y = 73.7^\circ$, $z = 61.3^\circ$
or $x = 7.07$, $y = 106^\circ$, $z = 28.7^\circ$
i $x = 49.8^\circ$, $y = 9.39$, $z = 37.0^\circ$
- 2 a $\angle ACB = 32.4^\circ$, $\angle ABC = 108^\circ$, $AC = 15.1$ cm
 $Syprina = 41.3$ cm²
b $\angle BAC = 41.5^\circ$, $\angle ABC = 28.5^\circ$, $AB = 9.65$ cm
 $Syprina = 15.7$ cm²



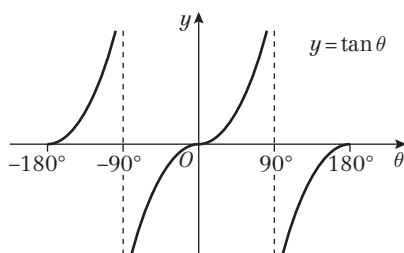
- 3 a 8 km b 060°
 4 107 km
 5 12 km
 6 a 5.44 b 7.95 c 36.8°
 7 a $AB + BC > AC \Rightarrow x + 6 > 7 \Rightarrow x > 1$;
 $AC + AB > BC \Rightarrow 11 > x + 2 \Rightarrow x < 9$
 b i $x = 6.08$ nga $x^2 = 37$
 $Syprina = 14.0\text{ cm}^2$
 ii $x = 7.23$ nga $x^2 - 4(\sqrt{2} - 1)x - (29 + 8\sqrt{2}) = 0$
 $Syprina = 13.1\text{ cm}^2$
 8 a $x = 4$ b 4.68 cm^2
 9 $AC = 1.93\text{ cm}$
 10 a $AC^2 = (2 - x)^2 + (x + 1)^2 - 2(2 - x)(x + 1) \cos 120^\circ$
 $= (4 - 4x + x^2) + (x^2 + 2x + 1) - 2(-x^2 + x + 2)(-\frac{1}{2})$
 $= x^2 - x + 7$
 b $\frac{1}{2}$
 11 $4\sqrt{10}$
 12 $AC = 1\frac{2}{3}\text{ cm}$ dhe $BC = 6\frac{1}{3}\text{ cm}$
 $Syprina = 5.05\text{ cm}^2$
 13 a 61.3° b 78.9 cm^2
 14 a $DAB = 136.3^\circ$, $BCD = 50.1^\circ$
 b 13.1 m^2
 c 5.15 m
 15 34.2 cm^2

Ushtrime 9F

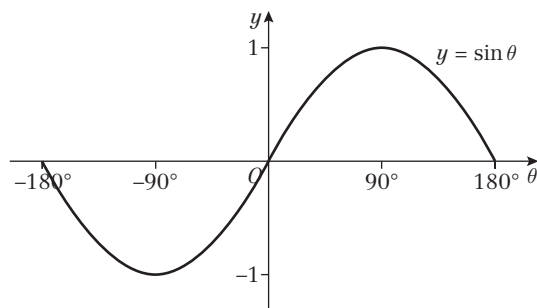
1



2



3



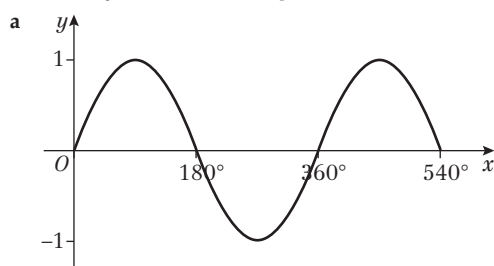
4

- a -30°
 b i -120° ii $-60^\circ, 120^\circ$
 c i 135° ii $-45^\circ, -135^\circ$

KAPITULLI 10

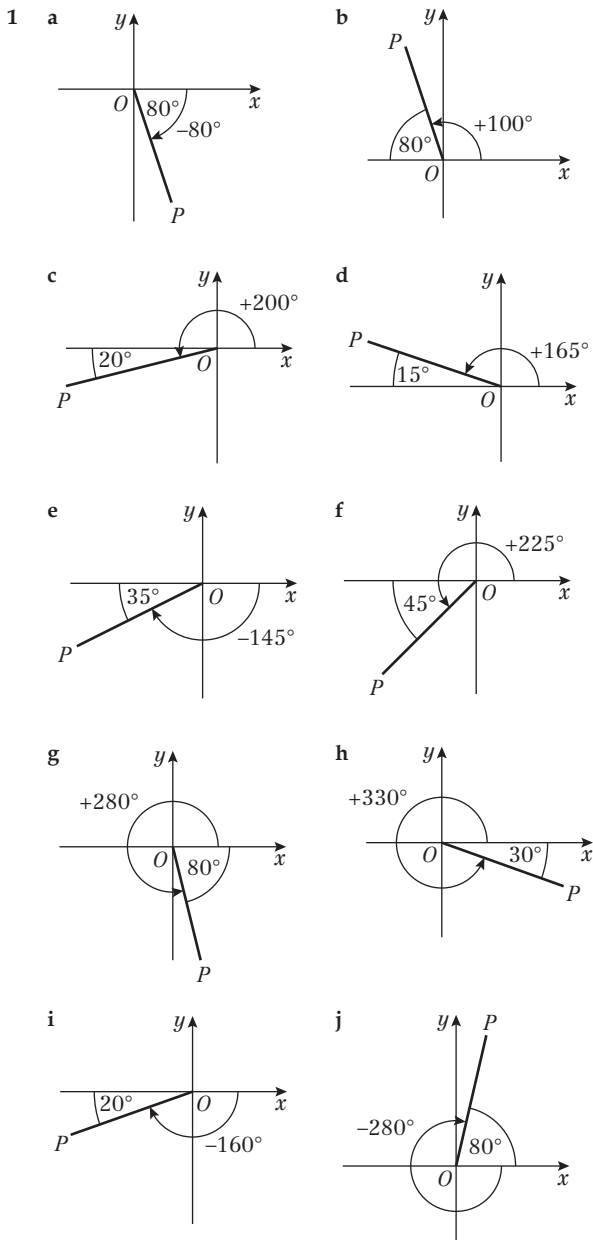
Kontrolli i njohurive të mëparshme

1



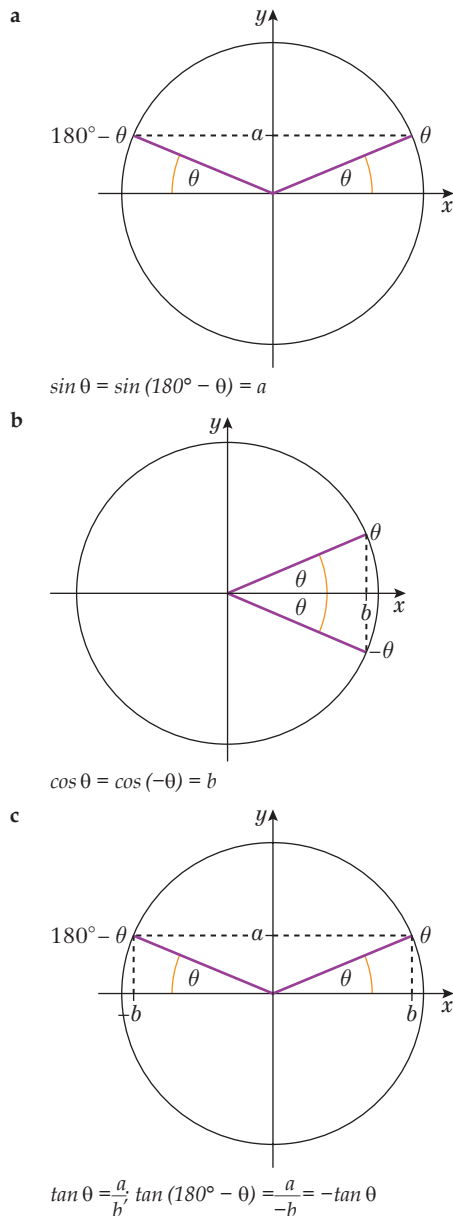
- b 4
 c $143.1^\circ, 396.9^\circ, 503.1^\circ$
 2 a 57.7° b 73.0°
 3 a $x = 11$ b $x = \frac{9}{4}$ c $x = -44.4^\circ$
 4 a $x = 1$ ose $x = 3$
 b $x = 1$ ose $x = -9$
 c $x = \frac{3 \pm \sqrt{65}}{4}$

Ushtrime 10A



- | | | |
|----------------------|--------------------|--------------------|
| 2 a I pari | b I dyti | c I dyti |
| d I treti | e I treti | |
| 3 a -1 | b 1 | c 0 |
| f 0 | g 0 | h 0 |
| 4 a $-\sin 60^\circ$ | b $-\sin 80^\circ$ | c $\sin 20^\circ$ |
| d $-\sin 60^\circ$ | e $\sin 80^\circ$ | f $-\cos 70^\circ$ |
| g $-\cos 80^\circ$ | h $\cos 50^\circ$ | i $-\cos 20^\circ$ |
| j $-\cos 5^\circ$ | k $-\tan 80^\circ$ | l $-\tan 35^\circ$ |
| m $-\tan 30^\circ$ | n $\tan 5^\circ$ | o $\tan 60^\circ$ |
| 5 a $-\sin \theta$ | b $-\sin \theta$ | c $-\sin \theta$ |
| d $\sin \theta$ | e $-\sin \theta$ | f $\sin \theta$ |
| g $-\sin \theta$ | h $-\sin \theta$ | i $\sin \theta$ |
| 6 a $-\cos \theta$ | b $-\cos \theta$ | c $\cos \theta$ |
| d $-\cos \theta$ | e $\cos \theta$ | f $-\cos \theta$ |
| g $-\tan \theta$ | h $-\tan \theta$ | i $\tan \theta$ |
| j $\tan \theta$ | k $-\tan \theta$ | l $\tan \theta$ |

Sfidë



Ushtrime 10B

- | | | | |
|--------------------------|-------------------------|-----------------|------------------------|
| 1 a $\frac{\sqrt{2}}{2}$ | b $-\frac{\sqrt{3}}{2}$ | c $\frac{1}{2}$ | d $\frac{\sqrt{3}}{2}$ |
| e $\frac{\sqrt{3}}{2}$ | f $-\frac{1}{2}$ | g $\frac{1}{2}$ | h $\frac{\sqrt{2}}{2}$ |
| i $\frac{\sqrt{3}}{2}$ | j $-\frac{\sqrt{2}}{2}$ | k -1 | l -1 |
| m $\frac{\sqrt{3}}{3}$ | n $-\sqrt{3}$ | o $\sqrt{3}$ | |

Sfidë

- | | | | |
|------------------------------------------------|------------------------------------|---------------------------|-------------------------------------|
| a i $\sqrt{3}$ | ii 2 | iii $\sqrt{2 + \sqrt{3}}$ | iv $\sqrt{2 + \sqrt{3}} - \sqrt{2}$ |
| b 15° | | | |
| c i $\frac{\sqrt{2 + \sqrt{3}} - \sqrt{2}}{2}$ | ii $\frac{\sqrt{2 + \sqrt{3}}}{2}$ | | |

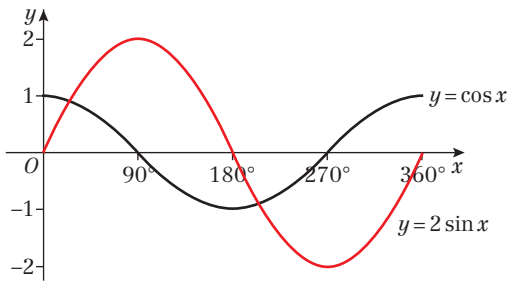


Ushtrime 10C

- 1 a $\sin^2 \frac{\theta}{2}$ b 5 c $-\cos^2 A$
 d $\cos \theta$ e $\tan x$ f $\tan 3A$
 g 4 h $\sin^2 \theta$ i 1
- 2 $\frac{1}{2}$
- 3 $3 \tan y$
- 4 a $1 - \sin^2 \theta$ b $\frac{\sin^2 \theta}{1 - \sin^2 \theta}$ c $\sin \theta$
 d $\frac{1 - \sin^2 \theta}{\sin \theta}$ e $1 - 2 \sin^2 \theta$
- 5 (Një shembull skicë e vërtetimit është dhënë)
- a Ana e majtë = $\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$
 $= 1 + 2 \sin \theta \cos \theta$
 $= \text{Ana e djathtë}$
- b Ana e majtë = $\frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} = \sin \theta \times \frac{\sin \theta}{\cos \theta}$
 $= \sin \theta \tan \theta = \text{Ana e djathtë}$
- c Ana e majtë = $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$
 $= \frac{1}{\sin x \cos x} = \text{Ana e djathtë}$
- d Ana e majtë = $\cos^2 A - (1 - \cos^2 A) = 2 \cos^2 A - 1$
 $= 2(1 - \sin^2 A) - 1 = 1 - 2 \sin^2 A = \text{Ana e djathtë}$
- e Ana e majtë = $(4 \sin^2 \theta - 4 \sin \theta \cos \theta + \cos^2 \theta)$
 $+ (\sin^2 \theta + 4 \sin \theta \cos \theta + \cos^2 \theta)$
 $= 5(\sin^2 \theta + \cos^2 \theta) = 5 = \text{Ana e djathtë}$
- f Ana e majtë = $2 - (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta)$
 $= 2(\sin^2 \theta + \cos^2 \theta) - (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta)$
 $= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$
 $= (\sin \theta + \cos \theta)^2 = \text{Ana e djathtë}$
- g Ana e majtë = $\sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y$
 $= \sin^2 x - \sin^2 y = \text{Ana e djathtë}$
- 6 a $\sin \theta = \frac{5}{13}, \cos \theta = \frac{12}{13}$
 b $\sin \theta = \frac{4}{5}, \tan \theta = \frac{4}{3}$
 c $\cos \theta = \frac{24}{25}, \tan \theta = \frac{7}{24}$
- 7 a $-\frac{\sqrt{5}}{3}$ b $-\frac{2\sqrt{5}}{5}$
- 8 a $-\frac{\sqrt{3}}{2}$ b $\frac{1}{2}$
- 9 a $-\frac{\sqrt{7}}{4}$ b $-\frac{\sqrt{7}}{3}$
- 10 a $x^2 + y^2 = 1$
 b $4x^2 + y^2 = 4$ (ose $x^2 + \frac{y^2}{4} = 1$)
 c $x^2 + y = 1$
 d $x^2 = y^2 (1 - x^2)$ (ose $x^2 + \frac{x^2}{y^2} = 1$)
 e $x^2 + y^2 = 2$ (ose $\frac{(x+y)^2}{4} + \frac{(x-y)^2}{4} = 1$)
- 11 a Nga teorema e kosinusit $\cos B = \frac{8^2 + 12^2 - 10^2}{2 \times 8 \times 12} = \frac{9}{16}$
 b $\frac{\sqrt{175}}{16}$

- 12 a Nga teorema e sinusit: $\sin Q = \frac{\sin 30^\circ}{6} \times 8 = \frac{2}{3}$
 b $\frac{\sqrt{5}}{3}$

Ushtrime 10D

- 1 a -63.4° b $116.6^\circ, 296.6^\circ$
 2 a 66.4° b $66.4^\circ, 113.6^\circ, 246.4^\circ, 293.6^\circ$
 3 a 270° b $60^\circ, 240^\circ$
 c $60^\circ, 300^\circ$ d $15^\circ, 165^\circ$
 e $140^\circ, 220^\circ$ f $135^\circ, 315^\circ$
 g $90^\circ, 270^\circ$ h $230^\circ, 310^\circ$
- 4 a $45.6^\circ, 134.4^\circ$ b $135^\circ, 225^\circ$
 c $132^\circ, 228^\circ$ d $229^\circ, 311^\circ$
 e $8.13^\circ, 188^\circ$ f $61.9^\circ, 242^\circ$
 g $105^\circ, 285^\circ$ h $41.8^\circ, 318^\circ$
- 5 a $30^\circ, 210^\circ$ b $135^\circ, 315^\circ$
 c $53.1^\circ, 233^\circ$ d $56.3^\circ, 236^\circ$
 e $54.7^\circ, 235^\circ$ f $148^\circ, 328^\circ$
- 6 a $-120^\circ, -60^\circ, 240^\circ, 300^\circ$ b $-171^\circ, -8.63^\circ$
 c $-144^\circ, 144^\circ$ d $-327^\circ, -32.9^\circ$
 e $150^\circ, 330^\circ, 510^\circ, 690^\circ$ f $251^\circ, 431^\circ$
- 7 a $\tan x$ duhet të jetë $\frac{2}{3}$
 b Ngritja në katror e të dyja anëve sjell zgjidhje të tjera.
 c $-146.3^\circ, 33.7^\circ$
- 8 a 
- b 2 c $26.6^\circ, 206.6^\circ$
- 9 $71.6^\circ, 108.4^\circ, 251.6^\circ, 288.4^\circ$
- 10 a $4 \sin^2 x - 3(1 - \sin^2 x) = 2$
 Rishkruaj dhe gjej $7 \sin^2 x = 5$
 b $57.7^\circ, 122.3^\circ, 237.7^\circ, 302.3^\circ$
- 11 a $2 \sin^2 x + 5(1 - \sin^2 x) = 1$
 Rishkruaj dhe gjej $3 \sin^2 x = 4$
 b $\sin x > 1$

Ushtrime 10F

- 1 a $60^\circ, 120^\circ, 240^\circ, 300^\circ$
 b $45^\circ, 135^\circ, 225^\circ, 315^\circ$
 c $0^\circ, 180^\circ, 199^\circ, 341^\circ, 360^\circ$
 d $77.0^\circ, 113^\circ, 257^\circ, 293^\circ$
 e $60^\circ, 300^\circ$
 f $204^\circ, 336^\circ$
 g $30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, 330^\circ$
- 2 a $\pm 45^\circ, \pm 135^\circ$ b $-180^\circ, -117^\circ, 0^\circ, 63.4^\circ, 180^\circ$
 c $\pm 114^\circ$ d $0^\circ, \pm 75.5^\circ, \pm 180^\circ$
- 3 a $72^\circ, 144^\circ$ b $0^\circ, 60^\circ$
 c Nuk ka zgjidhje në bashkësi.
- 4 a $\pm 41.8^\circ, \pm 138^\circ$ b $38.2^\circ, 142^\circ$
- 5 $60^\circ, 75.5^\circ, 284.5^\circ, 300^\circ$
- 6 $48.2^\circ, 131.8^\circ, 228.2^\circ, 311.8^\circ$
- 7 $2 \cos^2 x + \cos x - 6 = (2 \cos x - 3)(\cos x + 2)$
 Nuk ka zgjidhje për $\cos x = -2$ ose $\cos x = \frac{3}{2}$

- 8 a $1 - \sin^2 x = 2 - \sin x$
Rishkruaj dhe gjen $\sin^2 x - \sin x + 1 = 0$
b Ekuacioni nuk ka zgjidhje reale kur $b^2 - 4ac < 0$
9 a $p = 1, q = 5$
b $72.8^\circ, 129.0^\circ, 252.8^\circ, 309.0^\circ, 432.8^\circ, 489.0^\circ$

Sfidë

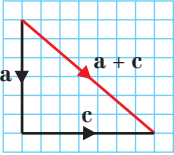
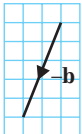
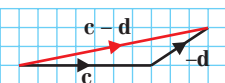
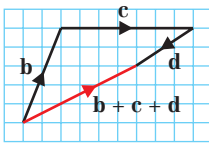
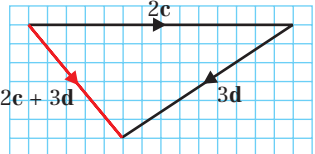
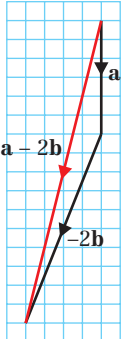
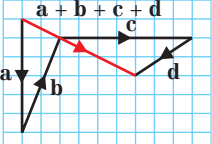
- 1 $-180^\circ, -60^\circ, 60^\circ, 180^\circ$
2 $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$

KAPITULLI 11

Kontrolli i njohurive të mëparshme

- 1 a $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ b $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ c $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$
2 a $\frac{7}{9}$ b $\frac{2}{9}$ c $\frac{7}{2}$
3 a 123.2° b 13.6 c 5.3 d 21.4°

Ushtrime 11A

- 1 a  b 
c  d 
e  f 
g 

- 2 a $2b$ b d c b
d $2b$ e $d + b$ f $d + b$
g $-2d$ h $-b$ i $2d + b$
j $-b + 2d$ k $-b + d$ l $-d - b$
3 a $2m$ b $2p$ c m
d m e $p + m$ f $p + m$
g $p + 2m$ h $p - m$ i $-m - p$
j $-2m + p$ k $-2p + m$ l $-m - 2p$
4 a $d - a$ b $a + b + c$
c $a + b - d$ d $a + b + c - d$

- 5 a $2a + 2b$ b $a + b$ c $b - a$
6 a b b $b - 3a$ c $a - b$
d $2a - b$
7 a $\overrightarrow{OB} = a + b$ b $\overrightarrow{OP} = \frac{5}{8}(a + b)$ c $\overrightarrow{AP} = \frac{5}{8}b - \frac{3}{8}a$
8 a Po ($\lambda = 2$) b Po ($\lambda = 4$) c Jo
d Po ($\lambda = -1$) e Po ($\lambda = -3$) f JO
9 a $i b - a$ ii $\frac{1}{2}a$ iii $\frac{1}{2}b$ iv $\frac{1}{2}b - \frac{1}{2}a$

- b $\overrightarrow{BC} = b - a$ $\overrightarrow{PQ} = \frac{1}{2}(b - a)$ pra PQ është paralele me BC .
10 a i $2b$ ii $a - b$
b $\overrightarrow{AB} = 2b$, $\overrightarrow{OC} = 3b$ pra AB është paralele me OC .
11 1.2

Ushtrime 11B

- 1 $v_1: 8i, \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ $v_2: 9i + 3j, \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ $v_3: -4i + 2j, \begin{pmatrix} -4 \\ 2 \end{pmatrix}$
 $v_4: 3i + 5j, \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $v_5: -3i - 2j, \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ $v_6: -5j, \begin{pmatrix} 0 \\ -5 \end{pmatrix}$
2 a $8i + 12j$ b $i + 1.5j$ c $-4i + j$
d $10i + j$ e $-2i + 11j$ f $-2i - 10j$
g $14i - 7j$ h $-8i + 9j$
3 a $\begin{pmatrix} 45 \\ 35 \end{pmatrix}$ b $\begin{pmatrix} 4 \\ 0.5 \end{pmatrix}$ c $\begin{pmatrix} 12 \\ 3 \end{pmatrix}$
d $\begin{pmatrix} -1 \\ 16 \end{pmatrix}$ e $\begin{pmatrix} -21 \\ -29 \end{pmatrix}$ f $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$
4 a $\lambda = 5$ b $\mu = \frac{3}{2}$
5 a $\lambda = \frac{1}{3}$ b $\mu = -1$
c $s = -1$ d $t = \frac{1}{17}$
6 $i - j$
7 a $\overrightarrow{AC} = 5i - 4j = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ b $\overrightarrow{AP} = 3i - \frac{13}{5}j = \begin{pmatrix} 3 \\ -\frac{13}{5} \end{pmatrix}$
c $\overrightarrow{OP} = 5i + \frac{8}{5}j = \begin{pmatrix} 5 \\ \frac{8}{5} \end{pmatrix}$
8 $j = 4, k = 11$
9 $p = 3, q = 2$
10 a $p = 5$ b $8i - 12j$

Ushtrime 11C

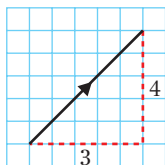
- 1 a 5 b 10 c 13
d 4.47 (3 sh.dh.) e 5.83 (3 sh.dh.) f 8.06 (3 sh.dh.)
g 5.83 (3 sh.dh.) h 4.12 (3 sh.dh.)
2 a $\sqrt{26}$ b $5\sqrt{2}$ c $\sqrt{101}$
3 a $\frac{14}{5b}$ b $\frac{1}{13(-12)}$
c $\frac{1-7}{25(24)}$ d $\frac{1}{\sqrt{10(-3)}}$
4 a 53.1° mbi b 53.1° nën
c 67.4° mbi d 63.4° mbi
5 a 149° djathtas b 29.7° djathtas
c 31.0° majtas d 104° majtas
6 a $\frac{15\sqrt{2}}{2}i + \frac{15\sqrt{2}}{2}j$ b $7.52i + 2.74j, \begin{pmatrix} 7.52 \\ 2.74 \end{pmatrix}$



$$c \quad 18.1i - 8.45j, \begin{pmatrix} 18.1 \\ -8.45 \end{pmatrix}$$

$$d \quad \frac{5\sqrt{3}}{2}i - 2.5j, \begin{pmatrix} 5\sqrt{3} \\ -2.5 \end{pmatrix}$$

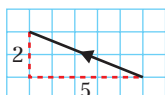
$$7 \quad a \quad |3i + 4j| = 5, 53.1^\circ \text{ mbi}$$



$$b \quad |2i - j| = \sqrt{5}, 26.6^\circ \text{ nën}$$



$$c \quad |-5i + 2j| = \sqrt{29}, 158.2^\circ \text{ mbi}$$



$$8 \quad k = \pm 6$$

$$9 \quad p = \pm 8, q = 6$$

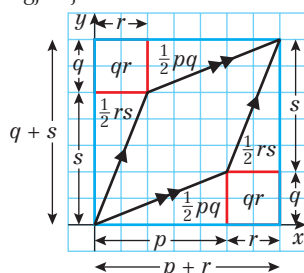
$$10 \quad a \quad 36.9^\circ \quad b \quad 33.7^\circ$$

$$c \quad 70.6^\circ$$

$$11 \quad a \quad 67.2^\circ \quad b \quad 19.0^\circ$$

Sfidë

Zgjidhje e mundshme"



Syprina e paralelogramit = syprina e drejtkëndëshit të madh - 2(syprina e drejtkëndëshit të vogël) - 2(syprina e trekëndëshit 1) - 2(syprina e trekëndëshit 2)

$$\text{Syprina e paralelogramit} = (p+r)(q+s) - 2qr - 2\left(\frac{1}{2}pq\right) - 2\left(\frac{1}{2}rs\right) = ps - qr$$

Ushtrime 11D

$$1 \quad a \quad i \quad \vec{OA} = 3i - j, \quad \vec{OB} = 4i + 5j, \quad \vec{OC} = -2i + 6j$$

$$ii \quad i + 6j \quad iii \quad -5i + 7j$$

$$b \quad i \quad \sqrt{40} = 2\sqrt{10} \quad ii \quad \sqrt{37} \quad iii \quad \sqrt{74}$$

$$2 \quad a \quad -i + 5j \text{ ose } \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$$b \quad i \quad 5 \quad ii \quad \sqrt{13} \quad iii \quad \sqrt{26}$$

$$3 \quad a \quad -i - 9j \text{ ose } \begin{pmatrix} -1 \\ -9 \end{pmatrix}$$

$$b \quad i \quad \sqrt{82} \quad ii \quad 5 \quad iii \quad \sqrt{61}$$

$$4 \quad a \quad -2a + 2b \quad b \quad -3a + 2b \quad c \quad -2a + b$$

$$5 \quad \begin{pmatrix} 7 \\ 9 \end{pmatrix} \text{ ose } \begin{pmatrix} 9 \\ 7 \end{pmatrix}$$

$$6 \quad a \quad 2i + 8j \quad b \quad 2\sqrt{17}$$

$$7 \quad \frac{3\sqrt{5}}{5}$$

Sfidë

$$\vec{OB} = 2i + 3j \text{ ose } \vec{OB} = \frac{46}{13}i + \frac{9}{13}j$$

Ushtrime 11E

$$1 \quad \vec{XY} = \mathbf{b} - \mathbf{a} \text{ dhe } \vec{YZ} = \mathbf{c} - \mathbf{b}, \text{ pra } \mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{b}.$$

Rrjedhimisht $\mathbf{a} + \mathbf{c} = 2\mathbf{b}$.

$$2 \quad a \quad i \quad 2r \quad ii \quad r$$

b Brinjët e trekëndëshit OAB janë sa dyfishi i gjatësisë së brinjëve të trekëndëshit PAQ dhe këndi A është i përbashkët te të dy SAS .

$$3 \quad a \quad \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

$$b \quad \vec{AN} = \frac{1}{3}(\mathbf{b} - \mathbf{a}) \quad \vec{AB} = \mathbf{b} - \mathbf{a}, \quad \vec{NB} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$$

$$AN : NB = 1 : 2.$$

$$4 \quad a \quad \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}$$

$$b \quad \vec{AP} = -\mathbf{a} + \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c} = \frac{2}{5}(\mathbf{c} - \mathbf{a}),$$

$$\vec{PC} = \mathbf{c} - \left(-\mathbf{a} + \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}\right) = \frac{3}{5}(\mathbf{c} - \mathbf{a}) \text{ pra } AP : PC = 2 : 3$$

$$5 \quad a \quad \sqrt{26} \quad b \quad 2\sqrt{2} \quad c \quad 3\sqrt{2}$$

$$d \quad \angle BAC = 56^\circ, \angle ABC = 34^\circ, \angle ACB = 90^\circ$$

$$6 \quad a \quad \vec{OR} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b},$$

$$\vec{OS} = 3 \vec{OR} = 3\left(\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}\right) = 2\mathbf{a} + \mathbf{b}$$

$$b \quad \vec{TP} = \vec{TO} + \vec{OP} = \mathbf{a} + \mathbf{b}, \quad \vec{PS} = \vec{OP} + \vec{OS} = -\mathbf{a} + 2\mathbf{a} + \mathbf{b} = \mathbf{a} + \mathbf{b}$$

\vec{TP} është paralel (dhe i barabartë) me \vec{PS} dhe ata kanë një pikë të përbashkët, P , pra T, P dhe S ndodhen në një vijë të drejtë.

Sfidë:

$$a \quad \vec{PR} = \mathbf{b} - \mathbf{a}, \quad \vec{PX} = j(\mathbf{b} - \mathbf{a}) = -j\mathbf{a} + j\mathbf{b}$$

$$b \quad \vec{ON} = \mathbf{a} + \frac{1}{2}\mathbf{b}, \quad \vec{PX} = -\mathbf{a} + k\left(\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = (k-1)\mathbf{a} + \frac{1}{2}k\mathbf{b}$$

c Koeficientet e \mathbf{a} dhe \mathbf{b} duhet të jenë të njëjtë në të dy shprehjet \vec{PX}

Koeficienti i \mathbf{a} : $k-1 = -j$; Koeficienti i \mathbf{b} : $j = \frac{1}{2}k$

$$d \quad \text{Duke zgjidhur njëherësh kemi } j = \frac{1}{3} \text{ dhe } k = \frac{2}{3}$$

$$e \quad \vec{PX} = \frac{1}{3}\vec{PR}$$

Nga simetria, $\vec{PX} = \vec{YR} = \vec{XY}$, pra ON dhe OM ndajnë PR në 3 pjesë të barabarta.

KAPITULLI 12

Kontrolli i njohurive të mëparshme

$$1 \quad a \quad 5 \quad b \quad -\frac{2}{3} \quad c \quad \frac{1}{3}$$

$$2 \quad a \quad x^{10} \quad b \quad x^{\frac{5}{3}} \quad c \quad x^{-1} \quad d \quad x^{\frac{2}{3}}$$

$$3 \quad a \quad y = \frac{1}{2}x - 2 \quad b \quad y = -\frac{1}{2}x + 8\frac{1}{2} \quad c \quad y = -\frac{1}{4}x + 7\frac{1}{2}$$

$$4 \quad y = -\frac{1}{2}x$$

Ushtrime 12A

- 1 a
- | | | | | | |
|--------------------------------------------|----|----|---|---|---|
| koordinata x | -1 | 0 | 1 | 2 | 3 |
| Vlerësimi për koeficientin këndor të vijës | -4 | -2 | 0 | 2 | 4 |
- b Gradienti = $2p - 2$ c 1
- 2 a $\sqrt{1 - 0.6^2} = \sqrt{0.64} = 0.8$
 b Gradienti = -0.75
 c i -1.21 (3 sh.dh.) ii -1 iii -0.859 (3 sh.dh.)
 d Kur pikat e tjera janë afër me A, atëherë koeficienti këndor tenton te -0.75 .
- 3 a i 7 ii 6.5 iii 6.1
 iv 6.01 v $h + 6$
 b Koeficientin këndor i tangjentes = 6
- 4 a i 9 ii 8.5 iii 8.1
 iv 8.01 v $8 + h$
 b Koeficientin këndor i tangjentes = 8

Ushtrime 12B

- 1 a $f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(2+h)^2 - 2^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{4h + h^2}{h} = \lim_{h \rightarrow 0} (4 + h) = 4$
- b $f'(-3) = \lim_{h \rightarrow 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h \rightarrow 0} \frac{(-3+h)^2 - 3^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{-6h + h^2}{h} = \lim_{h \rightarrow 0} (-6 + h) = -6$
- c $f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 - 0^2}{h} = \lim_{h \rightarrow 0} h = 0$
- d $f'(50) = \lim_{h \rightarrow 0} \frac{f(50+h) - f(50)}{h} = \lim_{h \rightarrow 0} \frac{(50+h)^2 - 50^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{100h + h^2}{h} = \lim_{h \rightarrow 0} (100 + h) = 100$
- 2 a $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = 2x$
- b Nëse $h \rightarrow 0$, $f'(x) = \lim_{h \rightarrow 0} (2x + h) = 2x$

- 3 a $g = \lim_{h \rightarrow 0} \frac{(-2+h)^3 - (-2)^3}{h}$
 $= \lim_{h \rightarrow 0} \frac{-8 + 3(-2)^2h + 3(-2)h^2 + h^3 + 8}{h}$
 $= \lim_{h \rightarrow 0} \frac{12h - 6h^2 + h^3}{h} = \lim_{h \rightarrow 0} (12 - 6h + h^2) = 12$
- b $g = 12$
- 4 a Koeficientin këndor i $AB = \frac{(-1+h)^3 - 5(-1+h) - 4}{(-1+h) - (-1)}$
 $= \frac{-1 + 3h - 3h^2 + h^3 + 5 - 5h - 4}{h}$
 $= \frac{h^3 - 3h^2 - 2h}{h} = h^2 - 3h - 2$
- b Koeficientin këndor = -2
- 5 $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{6(x+h) - 6x}{h} = \lim_{h \rightarrow 0} \frac{6h}{h} = 6$
- 6 $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{4(x+h)^2 - 4x^2}{h} = \lim_{h \rightarrow 0} \frac{8xh + 4h^2}{h}$
 $= \lim_{h \rightarrow 0} (8x + 4h) = 8x$
- 7 $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{a(x+h)^2 - ax^2}{h} = \lim_{h \rightarrow 0} \frac{(a-a)x^2 + 2axh + ah^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{2axh + ah^2}{h} = \lim_{h \rightarrow 0} (2ax + ah) = 2ax$

Sfidë

- a $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{x - (x+h)}{xh(x+h)} = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$
 $= \lim_{h \rightarrow 0} \frac{-1}{x^2 + xh} = \lim_{h \rightarrow 0} (2ax + ah) = 2ax$
- b $f'(x) = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2 + xh} = \frac{-1}{x^2 + 0} = -\frac{1}{x^2}$

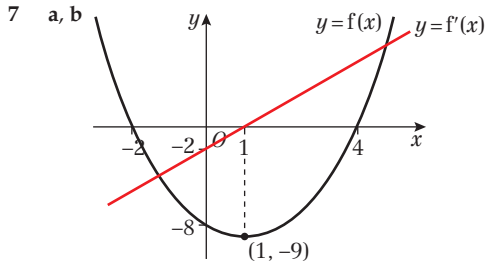
Ushtrime 12C

- 1 a $7x^6$ b $8x^7$ c $4x^3$ d $\frac{1}{3}x^{-3}$
 e $\frac{1}{4}x^{-3}$ f $\frac{1}{3}x^{-3}$ g $-3x^{-4}$ h $-4x^{-5}$
 i $-2x^{-3}$ j $-5x^{-6}$ k $-\frac{1}{2}x^{-3}$ l $-\frac{1}{3}x^{-3}$
 m $9x^8$ n $5x^4$ o $3x^2$ p $-2x^{-3}$
- 2 a $6x$ b $54x^8$ c $2x^3$ d $5x^{-3}$
 e $\frac{15}{2}x^{\frac{1}{2}}$ f $-10x^{-2}$ g $6x^2$ h $-\frac{1}{2x^5}$
 i x^{-3} j $\frac{15}{2}x^{\frac{1}{2}}$
- 3 a $\frac{3}{4}$ b $\frac{1}{2}$ c 3 d 2
- 4 $\frac{dy}{dx} = \frac{3\sqrt{x}}{22}$

Ushtrime 12D

- 1 a $4x - 6$ b $x + 12$ c $8x$ d $16x + 7$
 e $4 - 10x$
- 2 a 12 b 6 c 7 d $\frac{21}{2}$
 e -2 f 4
- 3 4, 0
- 4 $(-1, -8)$
- 5 1, -1
- 6 6, -4





- c At the turning point, the gradient of $y = f(x)$ is zero, i.e. $f'(x) = 0$.

Ushtrime 12E

- a $4x^3 - x^{-2}$ b $10x^4 - 6x^{-3}$ c $9x^3 - x^{-3}$
- a 0 b $11\frac{1}{2}$
- a $(\frac{21}{2}, -6\frac{1}{4})$ b $(4, -4)$ dhe $(2, 0)$
c $(16, -31)$ d $(\frac{1}{6}, 4)$ $(-\frac{1}{2}, -4)$
- a x^{-3} b $-6x^{-3}$ c $-x^{-4}$
d $\frac{4}{3}x^3 - 2x^2$ e $\frac{1}{2}x^{\frac{1}{2}} - 6x^{-4}$ f $\frac{1}{3}x^{\frac{2}{3}} - \frac{1}{2}x^{-2}$
g $-3x^{-2}$ h $3 + 6x^{-2}$ i $5x^3 + \frac{3}{2}x^{-3}$
j $3x^2 - 2x + 2$ k $12x^3 + 18x^2$ l $24x - 8 + 2x^{-2}$
- a 1 b $\frac{2}{9}$ c -4 d 4
- $-\frac{3}{4}\sqrt{2}$
- a $512 - 2304x + 4608x^2$
b $f'(x) \approx \frac{d}{dx}(512 - 2304x + 4608x^2)$
 $= -2304 + 2 \times 4608x$
 $= 9216x - 2304$

Ushtrime 12F

- a $y + 3x - 6 = 0$ b $4y - 3x - 4 = 0$
c $3y - 2x - 18 = 0$ d $y = x$
e $y = 12x + 14$ f $y = 16x - 22$
2 a $7y + x - 48 = 0$ b $17y + 2x - 212 = 0$
- $(\frac{12}{9}, \frac{18}{9})$
- $y = -x, 4y + x - 9 = 0; (-3, 3)$
- $y = -8x + 10, 8y - x - 145 = 0$
- $(-\frac{3}{4}, \frac{9}{8})$

Sfidë

L Ka ekuacion $y = 12x - 8$.

Ushtrime 12G

- a $x \geq -\frac{4}{3}$ b $x \leq \frac{2}{3}$ c $x \leq -2$
d $x \leq 2, x \geq 3$ e $x \in \mathbb{R}$ f $x \in \mathbb{R}$
g $x \geq 0$ h $x \geq 6$
- a $x \leq 4.5$ b $x \geq 2.5$
c $x \geq -1$ d $-1 \leq x \leq 2$
e $-3 \leq x \leq 3$ f $-5 \leq x < 0, 0 < x \leq 5$
g $0 < x \leq 9$ h $-2 \leq x \leq 0$
- f $(x) = -6x^2 - 3$
 $x^2 \geq 0$ për të gjithë $x \in \mathbb{R}$, pra $-6x^2 - 3 \leq 0$ për të gjithë $x \in \mathbb{R}$.
 $\therefore f(x)$ Është zvogëluar për të gjithë $x \in \mathbb{R}$.
- a Çdo $p \geq 2$
b Jo. Mund të jetë çdo $p \geq 2$.

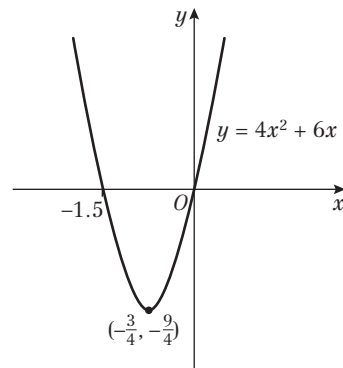
Ushtrime 12H

- a $24x + 3, 24$
b $15 - 3x^{-2}, 6x^{-3}$
c $\frac{9}{2}x^{-3} + 6x^{-3}, \frac{9}{4}x^{-3} - 18x^{-4}$
d $30x + 2, 30$
e $-3x^{-2} - 16x^{-3}, 6x^{-3} + 48x^{-4}$
- Përsheptimi $= \frac{3}{4}t^{\frac{1}{2}} + \frac{3}{2}t^{\frac{5}{2}}$
- $\frac{3}{2}$
- $-\frac{1}{2}$

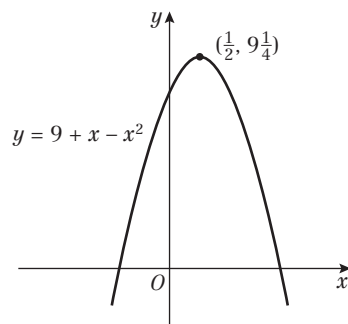
Ushtrime 12I

- a -28 b -17 c $-\frac{1}{5}$
- a 10 b 4 c 12.25
- a $(-\frac{3}{4}, -\frac{9}{4})$ minimum
b $(\frac{1}{6}, 9\frac{1}{4})$ maksimum
c $(-\frac{1}{3}, 1\frac{5}{27})$ maksimum, $(1, 0)$ minimum
d $(3, -18)$ minimum, $(-\frac{1}{3}, \frac{14}{27})$ maksimum
e $(1, 2)$ minimum, $(-1, -2)$ maksimum
f $(3, 27)$ minimum
g $(\frac{9}{4}, -\frac{9}{4})$ minimum
h $(2, -4\sqrt{2})$ minimum
i $(\sqrt{6}, -36)$ minimum, $(-\sqrt{6}, -36)$ minimum, $(0, 0)$ maksimum

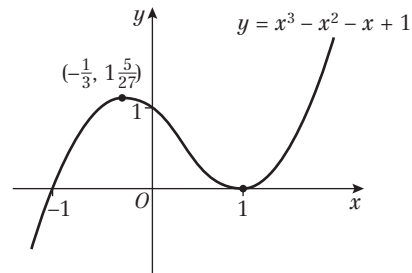
4 a

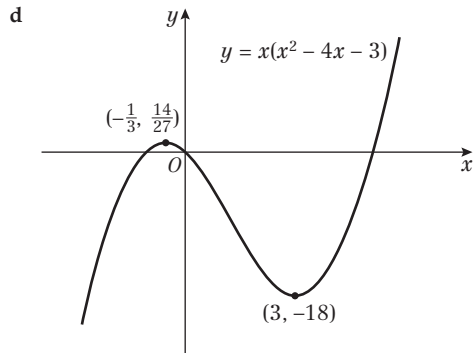


b

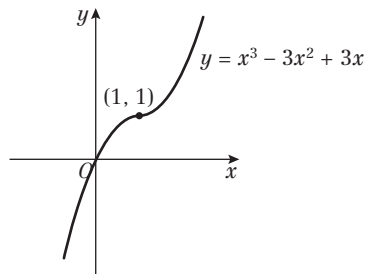


c



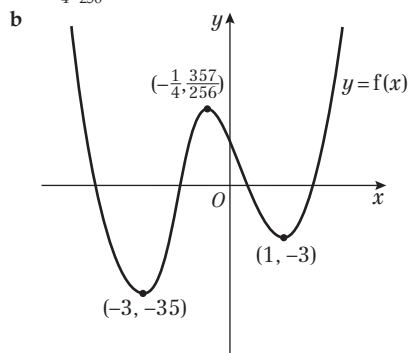


- 5 (1, 1) infleksion (Koefficientin këndor është pozitiv në një nga anët e pikës)



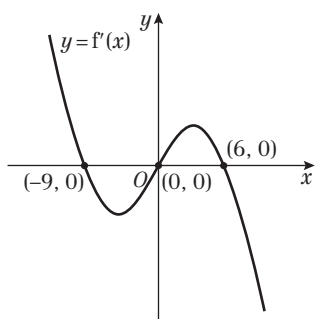
- 6 Vlera maksimale është 27; $f(x) :: 27$

- 7 a (1, -3): minimum, (-3, -35): minimum,
 $(-\frac{1}{4}, \frac{357}{256})$: maksimum

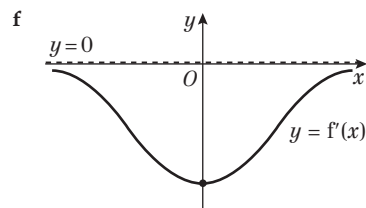
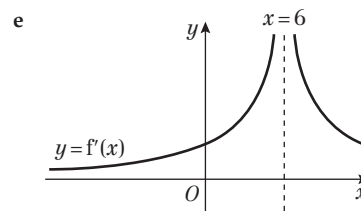
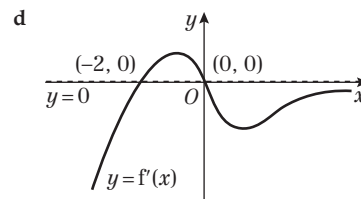
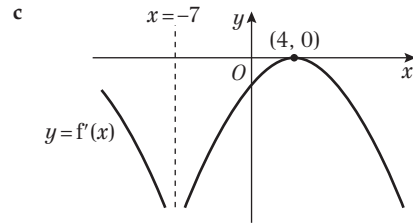
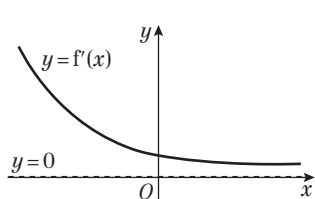


Ushtrime 12J

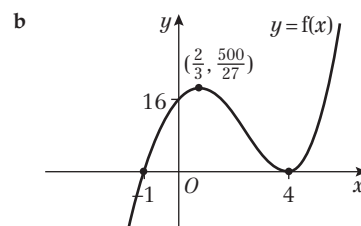
- 1 a



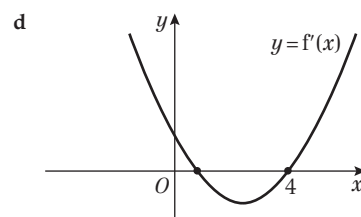
- b



- 2 a $f(x) = x^3 - 7x^2 + 8x + 16$
 $f'(x) = 3x^2 - 14x + 8 = (3x - 2)(x - 4)$



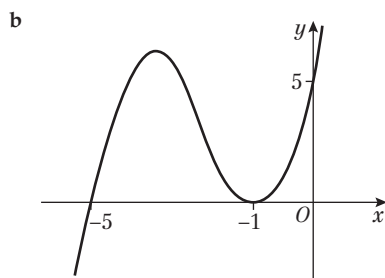
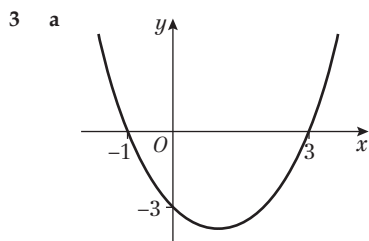
- c $(4, 0), (\frac{2}{3}, 0)$ dhe $(0, 8)$



KAPITULLI 13

Kontrolli i njohurive të mëparshme

- 1 a x^5 b $2x^3$ c $x^5 - \sqrt{x}$ d $x^{-3} + 4x$
 2 a $6x^2 + 3$ b $x - 1$ c $3x^2 + 2x$ d $-\frac{1}{x^2} - 3x^2$



Ushtrime 13A

- 1 a $y = \frac{1}{6}x^6 + c$ b $y = 2x^5 + c$
 c $y = x^{-1} + c$ d $y = 2x^{-2} + c$
 e $y = \frac{3}{5}x^5 + c$ f $y = \frac{8}{3}x^3 + c$
 g $y = -\frac{2}{7}x^7 + c$ h $y = 2x^3 + c$
 i $y = -10x^{-3} + c$ j $y = \frac{9}{2}x^4 + c$
 k $y = 3x^{12} + c$ l $y = 2x^{-7} + c$
 m $y = -9x^3 + c$ n $y = -5x + c$
 o $y = 3x^2 + c$ p $y = \frac{10}{3}x^{0.6} + c$
 2 a $y = \frac{1}{4}x^4 - 3x^3 + 6x^{-1} + c$ b $y = x^4 + 3x^3 + x^{-1} + c$
 c $y = 4x + 4x^{-3} + 4x^3 + c$ d $y = 3x^3 - 2x^5 - \frac{1}{2}x^{-2} + c$
 e $y = 4x^{-3} - 3x + 4x^2 + c$ f $y = x^5 + 2x^{-3} + 3x^{-4} + c$
 3 a $f(x) = 6x^2 - 3x^{-3} + 5x + c$ b $f(x) = x^6 - x^{-6} + x^{\frac{1}{6}} + c$
 c $f(x) = x^3 + x^{-3} + c$ d $f(x) = 2x^5 - 4x^{-2} + c$
 e $f(x) = 3x^3 - 6x^{-3} + c$
 f $f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^3 + c$
 4 $y = \frac{4x^3}{3} + 6x^2 + 9x + c$
 5 $f(x) = -3x^{-1} + 4x^3 + \frac{x^2}{2} - 4x + c$

Sfidë

$$y = -\frac{12}{7x^2} - \frac{4}{5x^5} + \frac{3}{2x^2} + \frac{1}{x} + c$$

Ushtrime 13B

- 1 a $\frac{x^4}{4} + c$ b $\frac{x^8}{8} + c$
 c $-x^{-3} + c$ d $\frac{5x^3}{3} + c$

- 2 a $\frac{1}{5}x^5 + \frac{1}{2}x^4 + c$ b $\frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$
 c $2x^5 - x^3 + c$
 3 a $-4x^{-1} + 6x^3 + c$ b $-6x^{-1} - \frac{2}{3}x^3 + c$
 c $-4x^{-\frac{1}{3}} + \frac{x^3}{3} - 2x^3 + c$
 4 a $x^4 + x^{-3} + rx + c$ b $\frac{1}{2}x^2 + 2x^3 - 2x^{-3} + c$
 c $\frac{px^5}{5} + 2qx - 3x^{-1} + c$
 5 a $t^3 + t^{-1} + c$ b $\frac{2t^3}{3} + 6t^{\frac{1}{2}} + t + c$
 c $\frac{p}{4}t^4 + q^2t + pr^3t + c$
 6 a $x^2 - \frac{3}{x} + c$ b $\frac{4}{3}x^3 + 6x^2 + 9x + c$
 c $\frac{4}{5}x^5 + 2x^3 + c$
 7 a $\frac{1}{3}x^3 + 2x - \frac{1}{x} + c$ b $\frac{1}{2}x^2 + \frac{8}{3}x^3 + 4x + c$
 c $2x^3 + \frac{4}{3}x^3 + c$
 8 a $\frac{3}{5}x^5 - \frac{2}{x^2} + c$ b $-\frac{1}{x^2} - \frac{1}{x} + 3x + c$
 c $\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$ d $\frac{8}{5}x^5 + \frac{8}{3}x^3 + 2x^3 + c$
 e $3x + 2x^3 + 2x^3 + c$ f $\frac{2}{5}x^5 + 3x^2 + 6x^3 + c$
 9 a $\frac{A}{x} - 3x + c$ b $\frac{2}{3}\sqrt{p}x^3 - \frac{1}{x^2} + c$
 c $-\frac{p}{x} + \frac{2qx^3}{3} + rx + c$
 10 $-\frac{6}{x} + \frac{8x^3}{3} - \frac{3x^2}{2} + 2x + c$
 11 $2x^4 + 3x^2 - 6x^3 + c$
 12 a $(2 + 5\sqrt{x})^2 = 4 + 10\sqrt{x} + 10\sqrt{x} + 25x = 4 + 20\sqrt{x} + 25x$
 b $4x + \frac{40x^3}{3} + \frac{25x^2}{2} + c$
 13 $\frac{x^6}{2} - 8x^3 + c$
 14 $p = -4, q = -2.5$
 15 a $1024 - 5120x + 11520x^2$
 b $1024x - 2560x^2 + 3840x^3 + c$

Ushtrime 13C

- 1 a $y = x^3 + x^2 - 2$ b $y = x^4 - \frac{1}{x^2} + 3x + 1$
 c $y = \frac{2}{3}x^3 + \frac{1}{12}x^3 + \frac{1}{3}$ d $y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$
 e $y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$ f $y = \frac{2}{5}x^3 + 6x^3 + 1$
 2 $f(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$
 3 $y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$
 4 $f(x) = 3x^3 + 2x^2 - 3x - 2$
 5 $y = 6x^3 - \frac{4x^5}{5} + \frac{118}{5}$
 6 a $p = \frac{1}{2}, q = 1$ b $y = 4x^3 + \frac{5x^2}{2} - \frac{421}{2}$
 7 a $f(t) = 10t - \frac{5t^2}{2}$ b $\frac{7}{2}$
 8 a $f(t) = -4.9t^2 + 35$ b 23.975 m
 c 35 m d 2.67 sekonda
 e p.sh. toka është e sheshtë.

Sfidë

- 1 $f_2(x) = \frac{x^3}{3}$; $f_4(x) = \frac{x^4}{12}$ b $\frac{x^{n+1}}{3 \times 4 \times 5 \times \dots \times (n+1)}$
- 2 $f_2(x) = x + 1$; $f_3(x) = \frac{1}{2}x^2 + x + 1$; $f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

Ushtrime 13D

- 1 a $152\frac{1}{4}$ b $48\frac{2}{5}$ c $5\frac{1}{3}$ d 2
- 2 a $5\frac{1}{4}$ b 10 c $11\frac{5}{6}$ d $60\frac{1}{2}$
- 3 a $16\frac{2}{3}$ b $46\frac{1}{2}$ c $\frac{11}{14}$ d $\frac{2}{2}$
- 4 $A = -7$ ose 4
- 5 28
- 6 $-8 + 8\sqrt{3}$
- 7 $k = \frac{25}{4}$
- 8 450 m

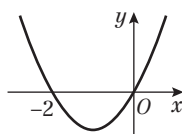
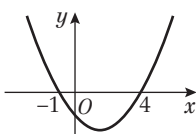
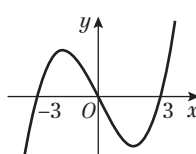
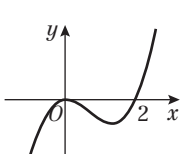
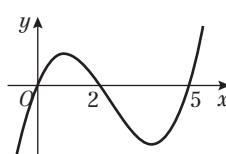
Sfidë

$$k = 2$$

Ushtrime 13E

- 1 a 22 b $36\frac{2}{3}$ c $48\frac{8}{15}$ d 6
- 2 4 3 6 4 $10\frac{2}{5}$
- 5 $21\frac{1}{3}$ 6 $\frac{4}{81}$ 7 $k = 2$
- 8 a $(-1, 0)$ dhe $(3, 0)$ b $10\frac{2}{3}$
- 9 $1\frac{1}{3}$

Ushtrime 13F

- 1 a $1\frac{1}{3}$ b $20\frac{5}{6}$
- 
- 
- c $40\frac{1}{2}$ d $1\frac{1}{3}$
- 
- 
- e $21\frac{1}{12}$
- 
- 2 a $(-3, 0)$ dhe $(2, 0)$ b $21\frac{1}{12}$
- 3 a $f(-3) = 0$
- b $f(x) = (x+3)(-x^2+7x-10)$
- c $f(x) = (x+3)(x-5)(2-x)$
- d $(-3, 0)$, $(2, 0)$ dhe $(5, 0)$
- e $143\frac{5}{6}$

Sfidë

- 1 a $4\frac{1}{2}$ b 9 c $\frac{9a}{2}$ d $4\frac{1}{2}$ e $\frac{9}{2a}$

- 2 a B e ka koordinatën x të barabartë me 1.

$$\int_0^1 (x^3 + x^2 - 2x) dx = \left[\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 \right]_0^1$$

$$= \frac{1}{4} + \frac{1}{3} - 1 = -\frac{5}{12}$$

Pra syprina nën boshtin x është $\frac{5}{12}$
syprina mbi x është

$$\left(\frac{1}{4}0^4 + \frac{1}{3}0^3 - 0^2 \right) - \left(\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 \right) = \frac{5}{12}$$

Pra koordinata x e a kënaq

$$3x^4 + 4x^3 - 12x^2 + 5 = 0$$

Pastaj përdor teoremën faktor dy herë që të gjes

$$(x-1)^2(3x^2+10x+5)=0$$

b A ka koordinata $\left(\frac{-5+\sqrt{10}}{3}, \frac{-80+37\sqrt{10}}{27} \right)$

Rrënjët në 1 i korrespondojnë pikës B.

Rrënja $\frac{-5-\sqrt{10}}{3}$ jep një pikë në vijë në të majtë të -2

nën boshtin x, pra nuk mund të jetë A.

Ushtrime 13G

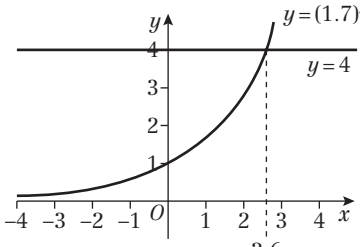
- 1 a $A(-2, 6)$, $B(2, 6)$ b $10\frac{2}{3}$
- 2 a $A(1, 3)$, $B(3, 3)$ b $1\frac{1}{3}$
- 3 $6\frac{2}{3}$
- 4 4.5
- 5 a $(2, 12)$ b $13\frac{1}{3}$
- 6 a $20\frac{5}{6}$ b $17\frac{1}{6}$
- 7 a, b Zëvendëso në ekuacion për y
- c $y = x - 4$ d $8\frac{3}{5}$
- 8 $3\frac{3}{8}$
- 9 a Zëvendëso $x = 4$ në gjithë ekuacionin
- b 7.2
- 10 a $21\frac{1}{3}$ b $2\frac{5}{9}$
- 11 a $(-1, 11)$ dhe $(3, 7)$ b $21\frac{1}{3}$

KAPITULLI 14

Kontrolli i njohurive të mëparshme

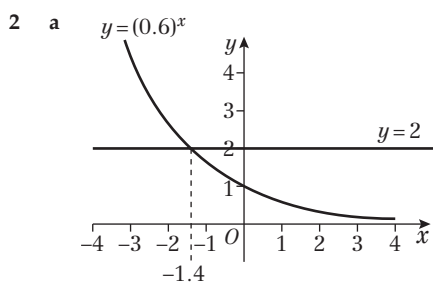
- 1 a 125 b $\frac{1}{3}$ c 32 d 49 e 1
- 2 a 6^6 b y^{21} c 2^6 d x^4
- 3 koeficienti këndor 1.5, ordinata në origjinë 4.1

Ushtrime 14A

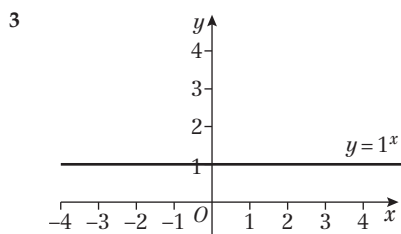
- 1 a
- 



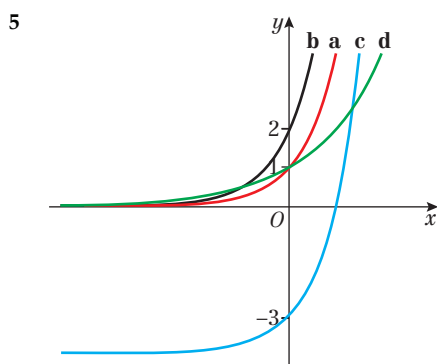
b $x \approx 2.6$



b $x \approx -1.4$



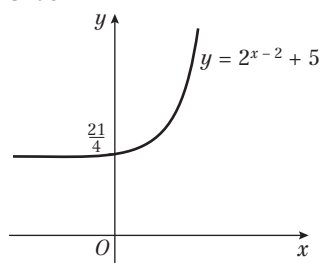
- 4 a I vërtetë, sepse $a^0 = 1$ për çdo a pozitive
 b Jo i vërtetë, për shembull kur $a = \frac{1}{2}$
 c I vërtetë, sepse kur a është pozitive, $a^x > 0$ për të gjitha vlerat e x



6 $k = 3, a = 2$

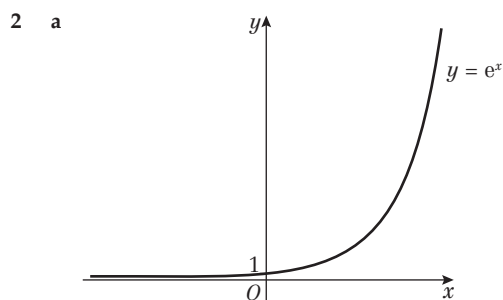
- 7 a Kur x rritet, y zvogëlohet
 b $p = 1.2, q = 0.2$

Sfidë



Ushtrime 14B

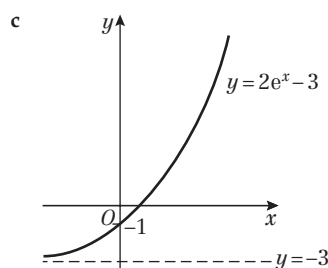
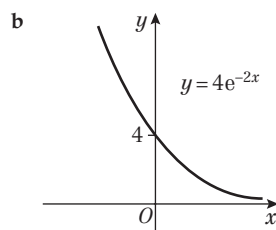
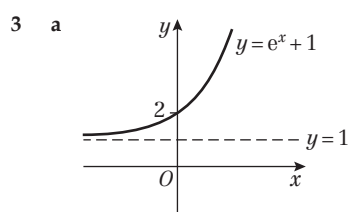
1 a 2.71828 b 54.59815 c 0.00004 d 1.22140

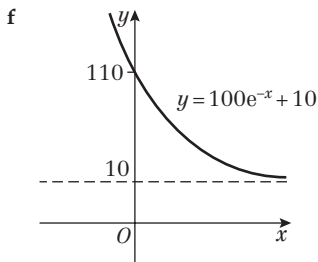
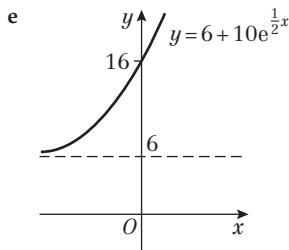
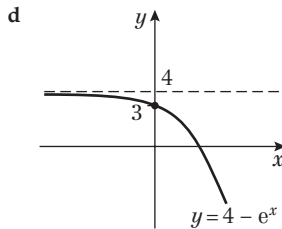


b Përgjigjet e nxënësit

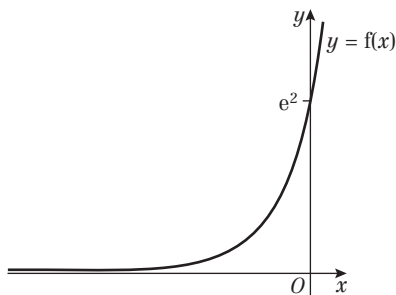
c $e = 2.71828...$

$e^3 = 20.08553...$





- 4 a $A = 1, C = 5, b$ është pozitiv
b $A = 4, C = 0, b$ është negativ
c $A = 6, C = 2, b$ është pozitiv
- 5 $A = e^2, b = 3$



- 6 a $6e^{6x}$ b $-\frac{1}{3}e^{-\frac{1}{3}x}$ c $14e^{2x}$
d $2e^{0.4x}$ e $3e^{3x} + 2e^x$ f $2e^{2x} + e^x$
- 7 a $3e^6$ b 3 c $3e^{-1.5}$
- 8 $f'(x) = 0.2e^{0.2x}$

Koeficientin këndor i tangjentes kur $x = 5$ është $f'(5) = 0.2e^1 = 0.2e$.
Ekuacioni i tangjentes rrjedhimisht është $y = (0.2e)x + c$.
Në $(5, e)$, $e = 0.2e \times 5 + c$, pra $c = 0$ dhe kur $x = 0, y = 0$.

Ushtrime 14D

- 1 a $\log_4 256 = 4$ b $\log_{\frac{1}{3}} 1 = -2$
c $\log_{10} 1\,000\,000 = 6$ d $\log_{11} 11 = 1$
e $\log_{0.2} 0.008 = 3$
- 2 a $2^4 = 16$ b $5^2 = 25$
c $9^2 = 81$ d $5^{-1} = 0.2$
e $10^5 = 100\,000$
- 3 a 3 b 2 c 7 d 1

- e 6 f $\frac{1}{2}$ g -1 h -2
i 10 j -2
4 a 625 b 9 c 7 d 9
e 20 f 2
5 a 2.475 b 2.173 c 3.009 d 1.099
6 a $5 = \log_2 32 < \log_2 50 < \log_2 64 = 6$
b 5.644
7 a i 1 ii 1 iii 1 b $a^1 \equiv a$
8 a i 0 ii 0 iii 0 b $a^0 \equiv 1$

Ushtrime 14E

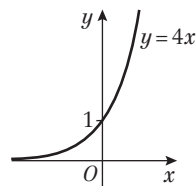
- 1 a $\log_2 21$ b $\log_2 9$ c $\log_5 80$
d $\log_6 \left(\frac{64}{81}\right)$ e $\log_{10} 120$
- 2 a $\log_2 8 = 3$ b $\log_6 36 = 2$ c $\log_{12} 144 = 2$
d $\log_8 2 = \frac{1}{3}$ e $\log_{10} \frac{1}{10} = -1$
- 3 a $3\log_a x + 4\log_a y + \log_a z$
b $5\log_a x - 2\log_a y$
c $2 + 2\log_a x$
d $\log_a x - \frac{1}{2}\log_a y - \log_a z$
e $\frac{1}{2} + \frac{1}{2}\log_a x$
- 4 a $\frac{4}{3}$ b $\frac{1}{18}$ c $\sqrt[3]{30}$ d 2
- 5 a $\log_3(x+1) - 2\log_3(x-1) = 1$
 $\log_3 \left(\frac{x+1}{(x-1)^2}\right) = 1$
 $\frac{x+1}{(x-1)^2} = 3$
 $x+1 = 3(x-1)^2$
 $x+1 = 3(x^2 - 2x + 1)$
 $3x^2 - 7x + 2 = 0$
b $x = 2$
- 6 a $a = 9, b = 4$

Sfidë

$\log_a x = m$ dhe $\log_a y = n$
 $x = a^m$ dhe $y = a^n$
 $x \div y = a^m \div a^n = a^{m-n}$
 $\log_a \left(\frac{x}{y}\right) = m - n = \log_a x - \log_a y$

Ushtrime 14F

- 1 a 6.23 b 2.10 c 0.431
d 1.66 e -3.22 f 1.31
g 1.25 h -1.73
- 2 a 0, 2.32 b 1.26, 2.18 c 1.21
d 0.631 e 0.565, 0.712 f 0
g 2 h -1
- 3 a 5.92 b 3.2
- 4 a (0, 1)



- b $\frac{1}{2\sqrt{2}}$
5 a 0.7565 b 7.9248 c 0.2966



Ushtrime 14G

- 1 a $\ln 6$ b $\frac{1}{2}\ln 11$ c $3 - \ln 20$
 d $\frac{1}{4}\ln\left(\frac{1}{6}\right)$ e $\frac{1}{2}\ln 3 - 3$ f $5 - \ln 19$
- 2 a e^2 b $\frac{e}{4}$ c $\frac{1}{2}e^4 - \frac{3}{2}$
 d $\frac{1}{6}(e^3 + 2)$ e $18 - e^3$ f $2, 5$
- 3 a $\ln 2, \ln 6$ b $\frac{1}{2}\ln 2, 0$ c e^3, e^{-5}
 d $\ln 4, 0$ e $\ln 5, \ln\left(\frac{1}{6}\right)$ f e^6, e^{-2}
- 4 $\ln 3, 2\ln 2$
- 5 a $\frac{1}{8}(e^2 + 3)$ b $\frac{1}{5}(\ln 3 + 40)$ c $\frac{1}{5}\ln 7, 0$
 d e^3, e^{-1}
- 6 $\frac{1 + \ln 5}{4 + \ln 3}$

- 7 a Përqendrimi i fillimit të mjekimit në mg/l

b 4.91 mg/l

c $3 = 6e^{-\frac{t}{10}}$

$\frac{1}{2} = e^{-\frac{t}{10}}$

$\ln\left(\frac{1}{2}\right) = -\frac{t}{10}$

$t = -10 \ln\left(\frac{1}{2}\right) = 6.931\dots = 6 \text{ hours } 56 \text{ minutes}$

- 8 a $(0, 3 + \ln 4)$ b $(4 - e^{-3})$

Sfidë

Me qenë se $y = 2$ është një asimptotë, $C = 2$.

Duke zëvendësuar $(0, 5)$, gjendet $5 = Ae^0 + 2$, pra A është 3.

Duke zëvendësuar $(6, 10)$ gjendet $10 = 3e^{6B} + 2$.

Duke rishkruar këtë, kemi $B = \frac{1}{6}\ln\left(\frac{8}{3}\right)$.



